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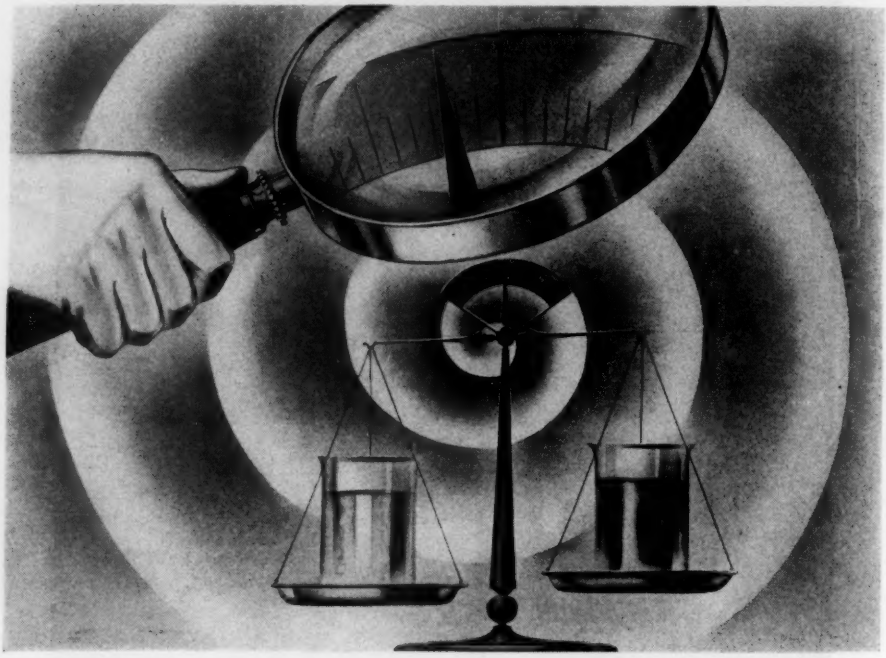

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STUDIES IN THE SENSITIVITY OF PHOTOGRAPHIC MATERIALS. II. EFFECTS ON THE SHAPE OF THE CHARACTERISTIC CURVES OF PHOTOGRAPHIC EMULSIONS.

BY

A. P. H. TRIVELLI,

Communication No. 1043 from the Kodak Research Laboratories.

INTRODUCTION.

In this paper are given data obtained during an investigation of the sensitivity of the grains of photographic emulsions. It is obvious that only tentative explanations of these data can be offered. They constitute an attempt to picture in a uniform totality the complexities which occur among sensitivity specks and development centers.

THE EFFECT OF HIGH- AND LOW-INTENSITY EXPOSURES UPON THE DEVELOPABILITY OF THE HIGHEST DENSITIES.

In a previous paper the author [1] emphasized the significance of the highest density of the characteristic curve as a factor in the determination of the sensitivity of a photographic emulsion. This had already been recognized in 1922 by Silberstein [2] in his first attempt to formulate an exposure theory of photographic materials. It is, therefore, obvious why the highest densities were investigated first.

If different areas of a photographic emulsion layer are exposed successively to increasing amounts of light, the densities produced after the emulsion has been developed for a definite time will reach a limit and then, on continued exposure, will in many, but not all, cases diminish. This highest density is called the maximum density, $D_{max.}$. Under the same conditions of illumination, this density increases only with prolonged development and not with prolonged exposure. If the

[1] Trivelli, A. P. H., "Studies in the Sensitivity of Photographic Materials," *Journ. Franklin Inst.*, **239**, 101, 1945.

[2] Silberstein, L., "Quantum Theory of Photographic Exposure," *Phil. Mag.*, **44**, 257, 1922.

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highest density at a definite development time does not diminish with continued exposure, it will be called a limiting density, $D_{\text{lim.}}$.

The results of the measurements of such high densities in photographic layers coated on commercial glass plates are, in general, unreliable, because the coatings are uneven, owing to the irregularity of the surface of the glass. The best, but not the most convenient way, to get the highest density values is from emulsions coated on *horizontal optically flat* glass plates. A much simpler method consists in using coatings on film which are far more even than those on commercial glass. Such emulsion layers give relatively good results if the highest densities are not more than $D \sim 2.80$. In nearly all the following investigations film coatings were used.

Maximum densities are obtained with continuous exposures, if solarization appears. Solarization is a high light-intensity effect. Low light-intensity exposures do not produce solarization. By gradually

TABLE I.

Sensitometric Results of Diminishing Light Intensity of Exposure.

Curve	I	$t_{\text{max.}}$			γ	$D_{\text{lim.}}$	$\gamma/D_{\text{lim.}}$
		Hr.	Min.	Sec.			
1	20		1	4	1.30	2.18	0.596
2	5		4	16	1.23	1.82	0.676
3	1.25		17	4	0.85	1.25	0.680
4	0.3125	1	8	16	0.48	0.66	0.727
5	0.078	4	33	4	0.26	0.32	0.813

diminishing the light intensities of exposures, maximum densities become limiting densities.

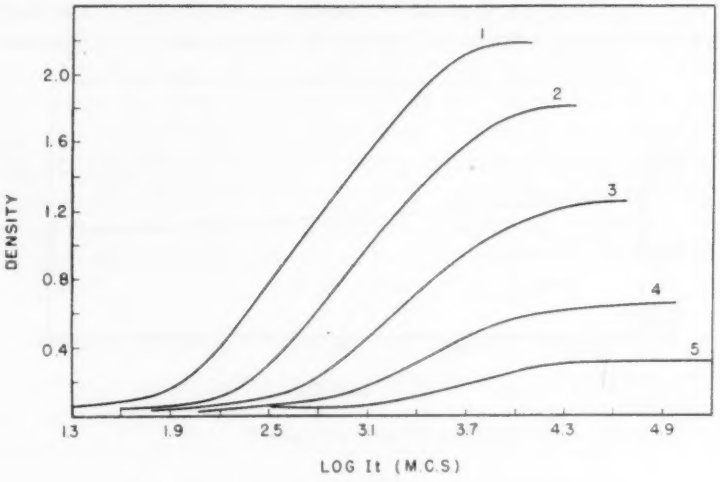
Maxima are generally regarded as the products of two opposing activities. The maximum density of the characteristic curve, therefore, may be regarded as the effect of an action (simple or complex) which produces developability of the grain and of another action which destroys it. Several investigations have been made which support the idea that the destruction of developability is the effect of the photolytically liberated bromine and bromine ions upon the development centers of the grains.[3]

The effects produced by diminishing the light intensity of the exposure upon the limiting densities of the characteristic curves were studied by Dr. J. H. Webb, of these Research Laboratories. At the request of the author, a fine-grain silver bromoiodide lantern-slide emulsion layer was exposed by Dr. Webb to a series of sensitometric step exposures, each succeeding step receiving the same total radiant

[3] See Mees, C. E. Kenneth, "The Theory of the Photographic Process," p. 247, The Macmillan Company, New York, 1942.

energy, It, but with different times of exposure, t, in minutes, and different light intensities, I, respectively. A number of different characteristic curves were obtained. With lower light intensities, the curves show, after development, lower limiting densities. In Table I are given the sensitometric data obtained for 3 minutes' development in a pyro-soda developer at 18° C. The characteristic curves are shown in Fig. 1. If the values of $\gamma/D_{\text{lim.}}$ are plotted on graph paper against the logarithm of the light intensities, I, a straight-line relation is obtained, following the equation:

$$\gamma/D_{\text{lim.}} = 0.71 - 0.08 \log I. \qquad (1)$$



FINE-GRAIN EMULSION EXPOSED TO DIFFERENT INTENSITIES WITH THE SAME DEVELOPMENT

FIG. 1.

This straight line, together with the observed data, is shown plotted on single logarithmic paper, in Fig. 2.

The decrease of the limiting densities with lower light-intensities of exposure is caused by a quite different action upon the latent image than that which occurs in the solarized region of the characteristic curve. The experiments of Weinland,[4] and of Webb and Evans [5] support the idea that the increased destructive effect on latent-image formation of low-intensity exposures is especially active during the initial stages of exposure. Thus, with a low-intensity exposure, the speck would be more likely to be broken up since the latent-image

[4] Weinland, C. E., "The Photographic Reciprocity Failure in the Underexposed Region," *Journ. Opt. Soc. Amer.*, **16**, 295, 1928.

[5] Webb, J. H., and Evans, C. H., "The Failure of the Photographic Reciprocity Law at Low Intensity," *Journ. Opt. Soc. Amer.*, **28**, 431, 1938.

speck forms slowly and exists for some time as a clump of only a relatively small number of silver atoms.

The effect of the low intensity holds for different development times. This is not the case with solarization. The author [6] showed that solarization occurs only with incomplete development. On prolonged development, the maximum density becomes a limiting density. This means that characteristic curves with maximum gammas have no maximum, but only limiting densities.

These observations were confirmed by Wilsey and Pritchard [6] and by Arens.[7] Solarization, therefore, must be regarded as a high-intensity effect produced by incomplete development.*

It is obvious that an exposure theory in itself does not explain a characteristic curve. Such a curve is the result of exposure and

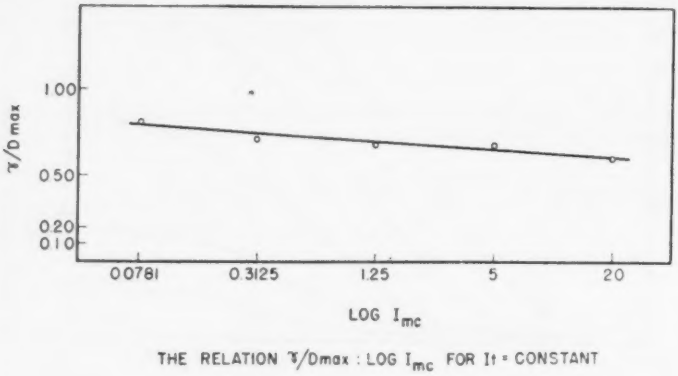


FIG. 2.

development. An explanation of the characteristic curve requires an exposure theory and a kinetic development theory combined. The principles of a kinetic development theory have been suggested by several investigators.

It is generally accepted that exposure [8] forms on the grains of the photographic emulsion centers from which development starts. With longer development times, the number of developed grains increases. At a certain development time shorter than infinite development, there

[6] Wilsey, R. B., and Pritchard, H. A., "A Comparison of X-ray and White-Light Exposures in Photographic Sensitometry," *Journ. Opt. Soc. Amer.*, **12**, 161, 1926. Trivelli, A. P. H., "Solarization und Entwicklung," *Zeitschr. wiss. Phot.*, **37**, 98, 1938.

[7] Arens, H., "Abhängigkeit der Solarization von der Entwicklungszeit," *Zeitschr. wiss. Phot.*, **33**, 106, 1934.

* H. J. Channon (*Brit. Journ., Phot.*, **74**, 340, 1927) found reversal effects at extremely low intensities and 15 months' exposure on Gem Medium Rapid Plates. To regard this reversal as solarization is questionable.

[8] The complications arising in the following pictures by considering internal latent images have been omitted for the sake of simplicity.

are a number of developable grains of which only a fraction develops. Development of a number of grains among a greater number of developable grains must be regarded as taking place according to the laws of probability in a way quantitatively similar to that in which developability is introduced by exposure.

It has been assumed that exposure makes a grain developable if the sensitivity speck reaches a definite size. This assumption in itself would be sufficient to account for an increase in the number of developed grains occurring with prolonged development. It would even hold for the limiting densities obtained by low-intensity exposures. The change of maximum density to a limiting density in the solarization region with prolonged development suggests that with high-intensity exposures there exist other development centers of smaller size. The chance that a smaller development center will initiate the reduction of the grain is less than that of a larger one. There is no reason why this concept of the existence of development centers of different sizes would not also hold for all exposure regions of the emulsion layer.[9] We may then assume that the smallest development center has the greatest chance of starting the reduction of the grain at maximum gamma.

This hypothetical picture must be borne in mind in evaluating the conclusions of many of the following investigations.

THE RELATION BETWEEN DENSITY AND THE NUMBER OF DEVELOPED GRAINS.

An increase in density takes place during development by progressive reduction of the light-affected silver halide grains, and, on prolonged development, by an increase in the number of developed grains.

Sheppard and Mees[10] found that the number of grains increases with the exposure, being approximately proportional to the density. In his quantum theory of exposure, Silberstein[11] assumes that in multigrain-layer emulsion coatings the density is proportional to the number of developed grains. This was confirmed by the agreement between observed and calculated densities in experiments with hundreds of characteristic curves.

Additional confirmation was obtained by Mr. R. P. Loveland and the author. A film was given a sensitometric step exposure. After 4 minutes' development in DK–50 developer at 18° C., development was checked in an acetic acid stop bath, and the film was thoroughly

[9] Even if the formation of such development centers is probably due to a haphazard distribution of sensitivity specks of different sizes.

[10] Sheppard, S. E., and Mees, C. E. Kenneth, "Investigations on the Theory of the Photographic Process," Longmans, Green, and Co., New York, Bombay, and Calcutta, p. 79, 1907.

[11] Silberstein, L., and Trivelli, A. P. H., "Quantum Theory of Exposure Tested Extensively on Photographic Emulsions," *Journ. Opt. Soc. Amer.*, **35**, 93, 1945.

washed. Alternate exposure steps were cut from the film and the developed silver was bleached with a chromic acid bath. The remaining silver halide grains were melted off and coated as single-grain-layer plates for grain counts. The remaining exposure steps were fixed in Kodak F-5 and the photographic densities measured in the usual way to give the characteristic density/log E curve. By graphical interpolation of the data so obtained it can be determined how many undeveloped grains per square centimeter are left at a given density. Subtracting these determinations from the total number of grains per square centimeter of the untreated film gives the relation between the number of developed grains, $N \times 10^6$, and the densities, D. The values for 4 minutes' development are given in Table II. The relation between

TABLE II.

Relation between Density and Number of Developed Grains.

D	$N_{obs.} \times 10^{-6}$	$N_{calc.} \times 10^{-6}$	$\Delta N \times 10^{-6}$
0.07	38	29	− 9
0.12	74	49	−25
0.24	94	99	+ 5
0.40	174	164	−10
0.62	251	255	+ 4
1.09	442	448	+ 6
1.23	510	505	− 5
1.44	585	592	+ 7
1.56	620	641	+21
1.58	672	649	−23

N and D is given by the linear equation:

$$N = 4 \times 10^8 D. \tag{2}$$

The deviations between observed and calculated values are shown in Table II. The data are plotted graphically in Fig. 3, showing a very good agreement between observed and calculated values. The same relation holds quantitatively for 3 and 12 minutes' development. This proportionality gives a general indication of which grains in an emulsion start development first. The information was obtained from a film coated with a silver bromide emulsion which had been sensitometrically exposed to monochromatic radiation of wavelength $\lambda = 426$ mμ and developed in a ferrous oxalate developer for 3 and 18 minutes at 18° C. The characteristic curves plotted have the following constants, t being time in minutes:

$$\text{for } t = 3 \text{ min.:} \quad D(y) = 1.19F(y),^* \tag{3}$$

$$\text{for } t = 18 \text{ min.:} \quad D(y) = 1.39F(y), \tag{4}$$

with their inflection points at the exposures $\log It = 0.67$, and 0.63,

* For tabulation of these functions see Reference 11.

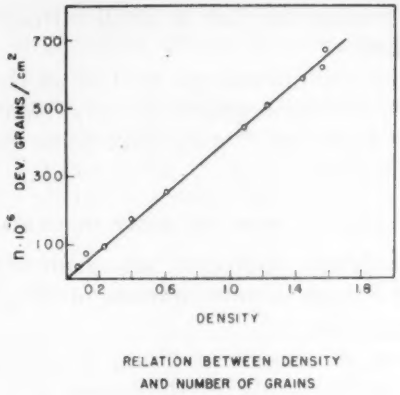


FIG. 3.

giving the intrinsic sensitivities $\epsilon_{3'} = 1.22 \times 10^{-4}$ and $\epsilon_{18'} = 1.33 \times 10^{-4}$. The percentage increase in density on prolonged development is found by plotting $(D_{18'} - D_{3'})/D_{3'}$ against the exposure axis. In this expression, $D_{3'}$ is the density at 3 minutes' development and $D_{18'}$ is the density produced by the same exposure at 18 minutes' development. The data are shown graphically in Fig. 4.

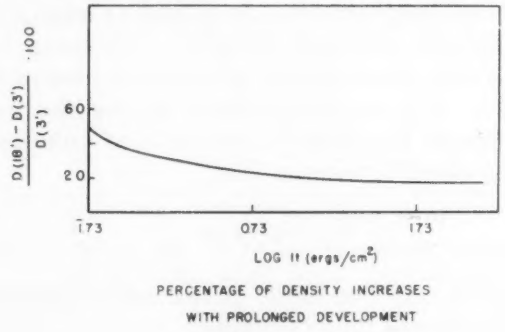


FIG. 4.

The curve in Fig. 4 shows that the less exposed parts which on the average contain larger developed grains have a higher percentage of increase in density than the parts of the film with longer exposure.[12] Therefore, the larger grains of an emulsion develop more quickly than the smaller grains. The generally observed greater increase in density at the lower exposures on prolonged development must then be regarded

[12] Measurements made by Mr. Loveland and the author show that at an exposure of $\log It = \bar{1}.69$, giving a density $D = 0.18$, the average grain size of the developed grains is $\bar{a} = 0.78\mu^2$, and at an exposure of $\log It = 1.91$, the corresponding values are $D = 1.65$ and $\bar{a} = 0.60\mu^2$, all data being taken from the same characteristic curve.

as due to the much greater number of smaller than larger grains in the photographic emulsion.

It was found under the microscope that in all these experiments the influence of partially reduced grains is very small. The possibility that "infectious development"[13] may play some part in these experiments is extremely small.

THE EFFECT OF SENSITIZING UPON THE SHAPE OF CHARACTERISTIC CURVES.

Photographic emulsions coated in one-grain-layer thickness show in all cases observed complete development of all grains with sufficient exposure and development.

Allowing sufficient development time, in photographic emulsions coated on film in a multigrain-layer thickness, 98 to 99 per cent. of all grains will develop with sufficient exposure on both sides of the film.

A much smaller number of developable grains were observed on the same film coatings if only the emulsion side was exposed, because of the shielding effect among the grains in the emulsion layer. This shielding effect can be measured quantitatively in the following experiment made by Mr. Loveland and the author. A sensitized silver bromoiodide emulsion with 2.56 moles per cent. of silver iodide was sensitometrically exposed on the emulsion side of the film in such a way that the complete characteristic curves from $D = 0$ to maximum density could be obtained for 2, 3, 4, 5, 8, and 12 minutes' development, respectively, in DK–50 developer at 20° C. By using Equation (2), it can be calculated that development of all the grains of the film gives a density $D = 2.70$. The shielding effect among the grains, however, gives, for the different development times, t, in minutes, the following percentages of developed grains at $D_{max.}$

t in minutes =	2	3	4	5	8	12
Percentage of grains developed =	53	56	57	61	63	68

After 12 minutes' development, at maximum density, 32 per cent. of all the grains were still undevelopable.

The shielding effect for the different grain sizes in a photographic emulsion can be measured graphically. In Fig. 5 are shown the curves plotted from a size-frequency analysis. Curve A is the distribution curve of the grains of the emulsion and Curve B is the distribution curve for the developed grains at maximum density after 5 minutes' development. The values representing the difference between Curves A and B, when plotted, give the distribution curve of the grains which were prevented from developing by the shielding effect during exposure. It should be noted that in this emulsion there are also a number of

[13] Stauffer, R. Eliot, Smith, W. F., and Trivelli, A. P. H., "The Influence of Photographic Developers Containing Hydrazines upon the Characteristic Curves of Photographic Materials," *Journ. Franklin Inst.*, **238**, 291, 1944.

very small grains, which do not become developable at all. The size-frequency curve of the shielded grains is an exponential one as shown by Curve C.

A study of the effect of the gelatin in sensitizing a photographic emulsion, therefore, is usually restricted to only a part of the grains of the emulsion without attempting to give any indication that results could not hold also for all the grains of the emulsion, if they all received an equal exposure.

The effect of sensitizing on the increase in the densities of the characteristic curves is investigated in relation to the sensitivity speck

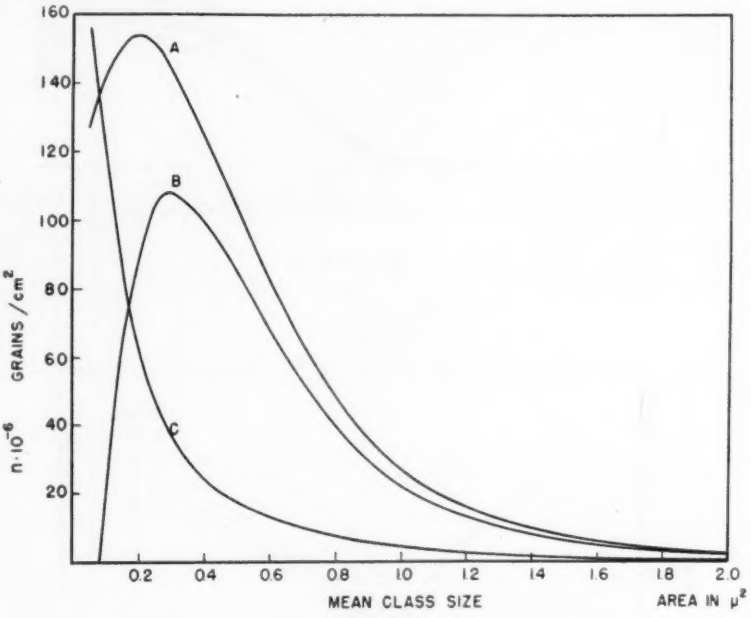


FIG. 5

and the development speck theories in the following way: An unsensitized silver bromide emulsion, A, with an average grain size $\bar{a} = 0.82\mu^2$, coated on film, gave, after exposure, development, and fog-correction,[14] the characteristic curves shown in Fig. 6 and 7. Figure 6 shows a family of curves for 2, 3, 4, 6, 9, and 18 minutes' development, respectively, in ferrous oxalate developer at 18° C., and Fig. 7 gives a family of curves for 2, 3, 4, 5, 8, and 12 minutes' development, respectively, in DK–50 developer at the same temperature. If this emulsion is sensitized with different gelatins, B, C, D, E, and F, the curves

[14] Trivelli, A. P. H., "Sensitivity Distribution among the Grains of Photographic Emulsions," *Journ. Franklin Inst.*, **239**, 101, 1945.

reproduced in Figs. 8 and 9 are produced. They were obtained with 6 minutes' and 5 minutes' development in FeOx and DK–50 developers, respectively. All these curves show an increase in density from $D = 0$ to $D_{max.}$ on treatment with the sensitizing gelatin. The same increase was noticed with all the other development times.

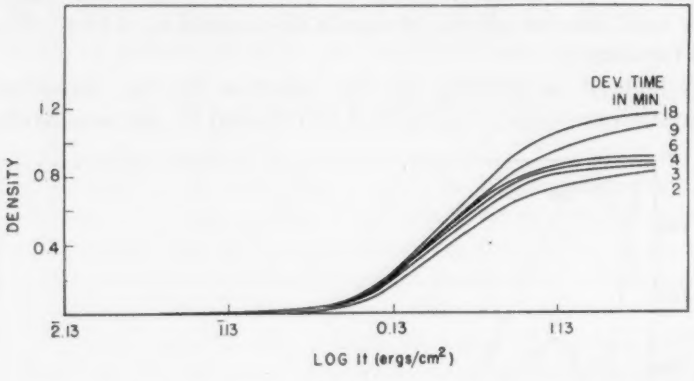


FIG. 6.

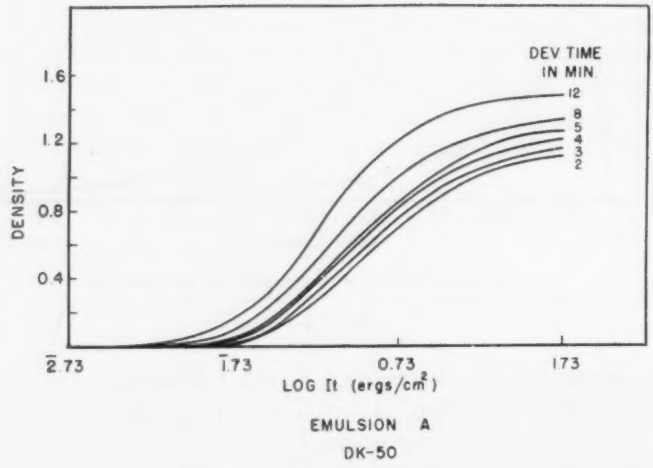


FIG. 7.

The increase in density produced by sensitizing may be due to different causes. Under the microscope, no partially reduced grains were found for 3 minutes' development or longer. The increase in density, therefore, must be due to an increase in the number of developed grains, together with an unavoidable change in the covering power of the reduced silver. The increase in the number of developable

grains on sensitizing can easily be demonstrated by photomicrographs of the grains taken at the maximum or limiting densities of the characteristic curves. They were made by the Photomicrography Department of these Research Laboratories and are shown in Fig. 10.

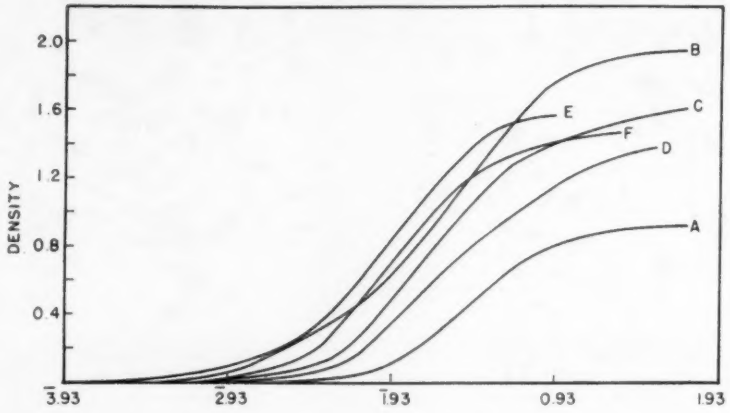


FIG. 8.

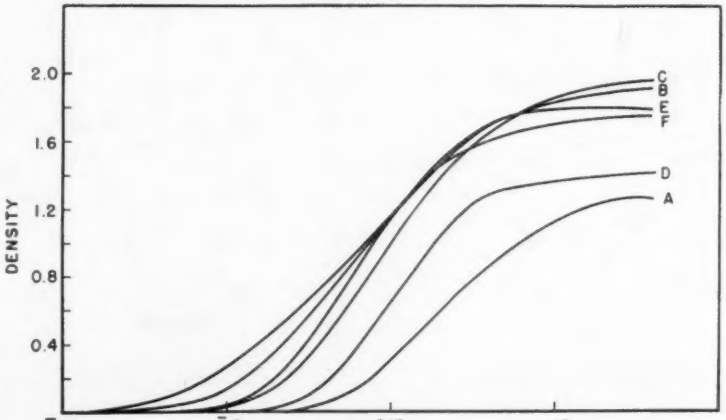


FIG. 9.

These pictures were obtained in the following way: The highest densities of equally exposed, developed, but not fixed films were removed from their support, melted in warm water, and coated in a one-grain-layer thickness on microscope slides. Two emulsions, *A* and *C*, were

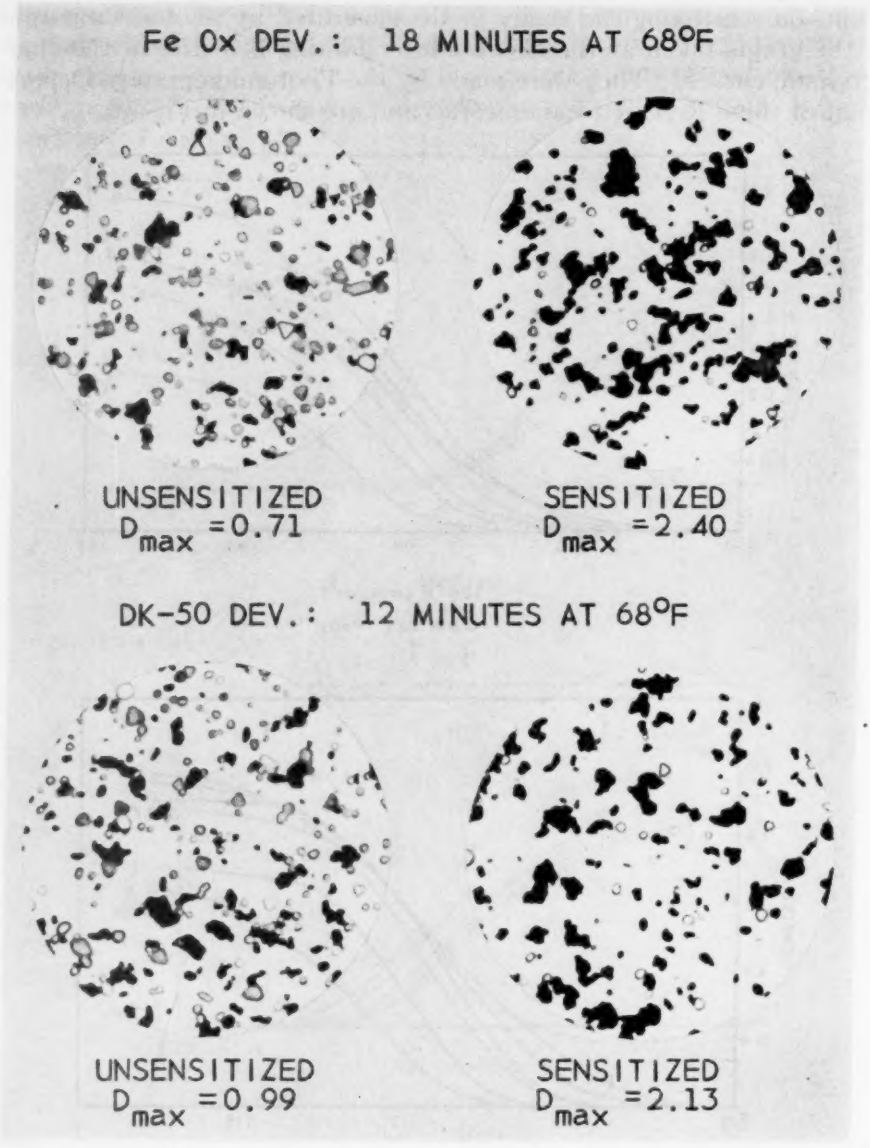


FIG. 10.

chosen for comparison. To obtain a very complete development without the introduction of too much fog, the films were developed after exposure for 18 minutes in ferrous oxalate and for 12 minutes in DK–50 developer at 20° C. The photomicrographs were made at 2,500 times enlargement of the grains to show the increase in the number of developed grains produced by sensitizing.

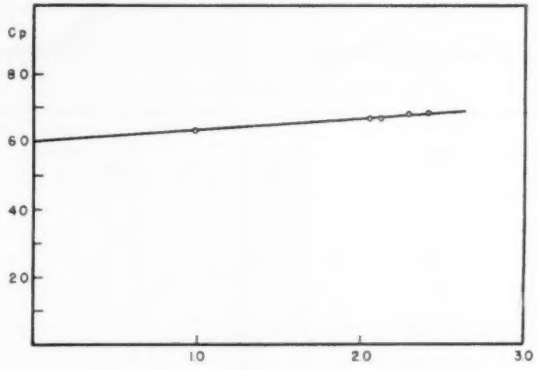
The magnitude of the change in the covering power, C_p, of the reduced silver by sensitizing was determined by Mr. B. Hutchins, of these Research Laboratories, for equally developed highest densities, which will be regarded as maximum densities for the sake of convenience. These determinations were made for all the film coatings mentioned. The relation is linear. Calculation by the least-squares method gives the equation:

$$C_p = 60 + 6.88D_{\max.}, \tag{5}$$

for development in ferrous oxalate, and

$$C_p = 60 + 3.19D_{\max.}, \tag{6}$$

for development in DK–50.



COVERING POWER AT DIFFERENT D_{max} OF THE EMULSIONS

DK-50

FIG. 11.

To show the agreement between the observed and the calculated values for Equation (6), the data were plotted graphically and are reproduced in Fig. 11.

The results show such small increases in the covering power with the increased density that they can be neglected in all the following discussion. The same thing holds true for all the other densities of the characteristic curves. It can be concluded that within the limits of experimental error the effect of sensitizing upon the densities is practically due to a proportional increase in the number of developed grains. The effect of sensitizing can be measured by the percentage of density increase over the whole characteristic curve. If D_u is the density of the unsensitized emulsion at a certain exposure with a definite time of development, and D_s is the density of the sensitized emulsion at the same exposure and with the same development time,

the values of $(D_s - D_u)/D_u$ can be plotted graphically against the logarithms of the exposures in ergs per square centimeter for a wavelength $\lambda = 426m\mu$. In this way the curves of Fig. 12 were obtained for 2, 6, 9, and 18 minutes' development, respectively, of the emulsions, *A*, *B*, *C*, *D*, *E*, and *F* in ferrous oxalate developer. It should be noted that Emulsion *A* is unsensitized.

Ferrous oxalate developer has the advantage over DK–50 developer in such an investigation that it shows less effect on the shifts of the inertia points of the characteristic curves towards smaller exposures as a result of sensitizing. At the same time the density increases are

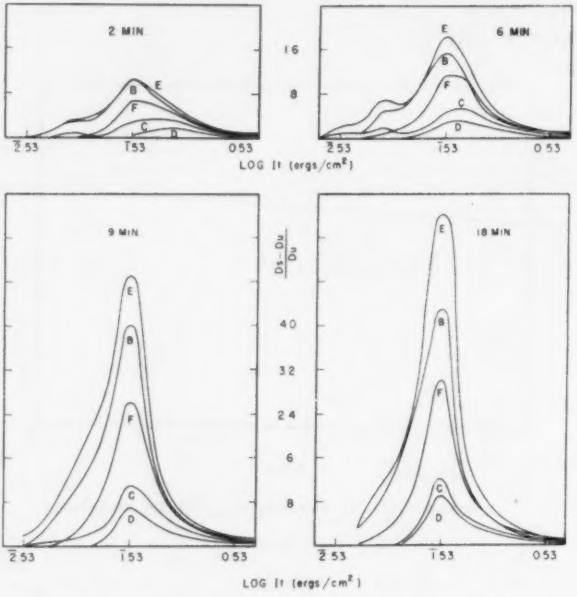


FIG. 12.

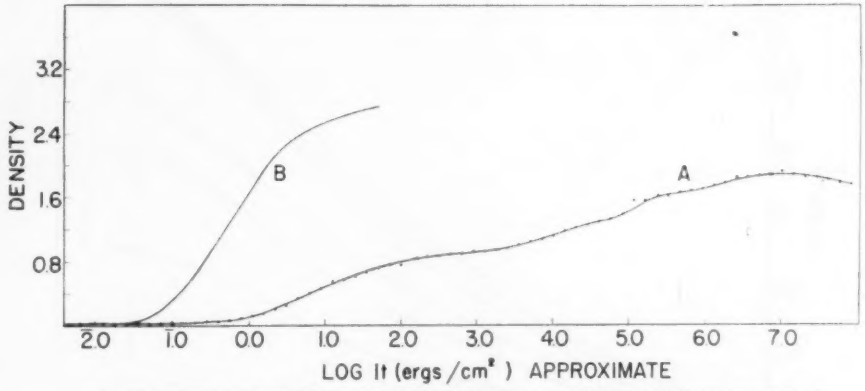
sufficiently large to produce a complete picture of the changes in the shapes of the characteristic curves. The following changes are noticeable.

From the curves of Fig. 12 it can be seen that the order of the magnitude of the sensitizing effects on the densities does not change with development time. Up to 6 minutes' development, several curves show two maxima, the smaller one of which disappears between 6 and 9 minutes on prolonged development. The sensitizing effects increase with longer development times. The greatest increase occurs between 6 and 9 minutes' development. The sensitivity maxima of the emulsions are reached with shorter development times at different exposures.

They all shift, however, to the same exposure, $\log It = \bar{1}.53$, with prolonged development.

Development in DK–50 gives a quite different picture. Owing to the greater shift of the inertia points of the characteristic curves of the sensitized emulsions developed in DK–50, the sensitivity maxima are shifted considerably to the regions of smaller exposures, approaching a value of $(D_s - D_u)/D_u = \infty$. These sensitivity curves show a diminishing of the sensitivity towards the longer exposures so that, on continued exposure, the sensitizing effect tends to disappear.

The curves of Fig. 12 show first an increase in the sensitizing effect with prolonged exposure until a maximum is reached. This may be explained by the same effects which occur with pre-exposures. Brentano and Baxter [15] demonstrated, for instance, the possibility of



CHARACTERISTIC CURVES OF EMULSIONS CONTAINING 2.56 MOLES % AgI

A. UNSENSITIZED

B. SENSITIZED

FIG. 13.

the existence of specks which are produced by pre-exposures, yielding centers which lie between sensitivity centers and development centers at a certain time of development.

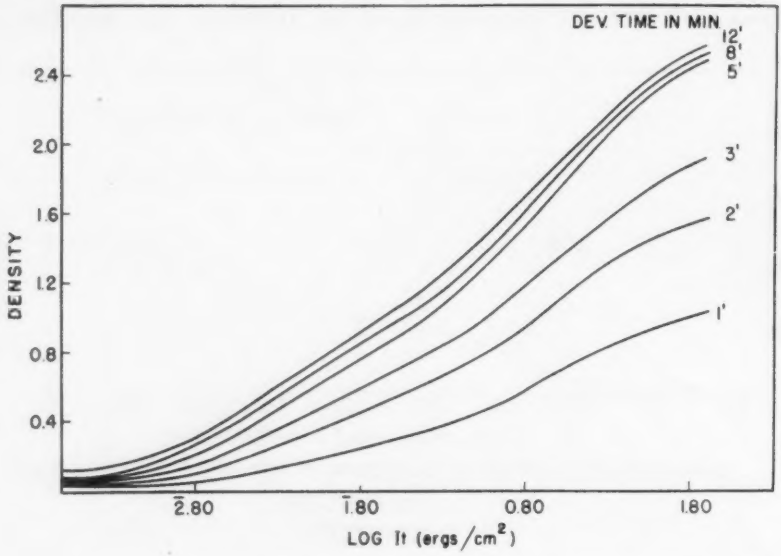
SENSITIVITY OF UNSENSITIZED EMULSIONS.

Even photographic emulsions which have been made with "non-sensitizing" gelatin show some sensitivity. They, like sensitized emulsions, have sensitivity specks. An interesting case of a difference in the sensitivity of the same emulsion with and without sensitizing was observed with a silver bromoiodide emulsion with 2.56 moles per cent.

[15] Brentano, J. C. M., and Baxter, S., "The Latent Photographic Image: Additivity of Part-Exposures and Microcrystalline Shape of Silver Halide Grains," *Trans. Faraday Soc.*, 36, 581, 1940.

of silver iodide. The developability of the emulsion was compared with and without sensitizing.

Mr. J. L. Tupper, of these Research Laboratories, found that the maximum density of films coated with the unsensitized emulsion could not be obtained with the usual sensitometer for 8 minutes' development in DK–50 at 20° C. Several trials were made with stronger light sources and longer exposures until at an exposure range of $1:10^8$ the required result was obtained. The characteristic curve obtained is shown as Curve A in Fig. 13. Sensitizing with a gelatin containing a sulfur sensitizer gives Curve B in the same figure. Curve A shows several irregular deviations from a smooth curve. Curves such as A



ANOMALOUS CHARACTERISTIC CURVES PRODUCED BY DK-50 DEVELOPMENT

FIG. 14.

have been found repeatedly. The experimental data are too few to warrant a general conclusion.

Curves A and B indicate the probable existence of different kinds of sensitivity specks having lower and higher potentialities of becoming development centers by exposure. The complex sensitivity distributions among the grains of photographic emulsions which sometimes occur [16] may also be explained by the existence of two different kinds of sensitivity specks with different potentialities of becoming development centers by exposure. This can be explained tentatively by

[16] Trivelli, A. P. H., "Sensitivity Distribution among the Grains of Photographic Emulsions," *Journ. Franklin Inst.*, **239**, 101, 1945.

differences in the depths of the energy levels which are connected with the tensions in the crystal structure around the specks.

ANOMALIES IN THE FORMATION OF CHARACTERISTIC CURVES.

Anybody investigating the sensitivity of photographic emulsions will sooner or later encounter characteristic curves with unexpected anomalies. Sometimes they are easily explained and sometimes they remain puzzles for the time being. A few of them will be mentioned.

(*a*) In Fig. 14 are shown the characteristic curves of a converted silver bromoiodide emulsion coated on film, containing 6.18 moles per cent. of silver iodide and having an average grain size $\bar{a} = 0.17\mu^2$, and a

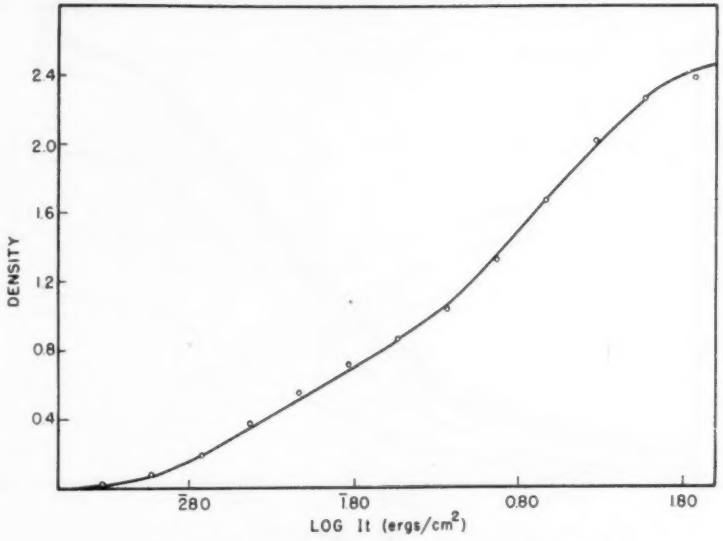


FIG. 15.

dispersion of grain sizes $\sigma = 0.28\mu^2$, which is relatively high, so that the uniformity of grain sizes $\bar{a}/\sigma = 0.60$ is low. Such low uniformities in grain sizes often occur in mixed emulsions, but this emulsion is a single one. The shape of these curves, in which the breaks in the curve repeat themselves over a considerable range of development times from 1 to 12 minutes in DK–50 developer at 20° C., dividing the curves into two approximately straight lines, cannot be explained therefore as the result of a binary mixture. Such shapes can, however, be explained satisfactorily by assuming that the emulsion contains two sensitivity distributions among its grains.

In Fig. 15 are shown the data from an analysis of the curve obtained with 5 minutes' development. This curve can be represented by the

combination of a two-quanta and a one-quantum curve, following the equation:[17]

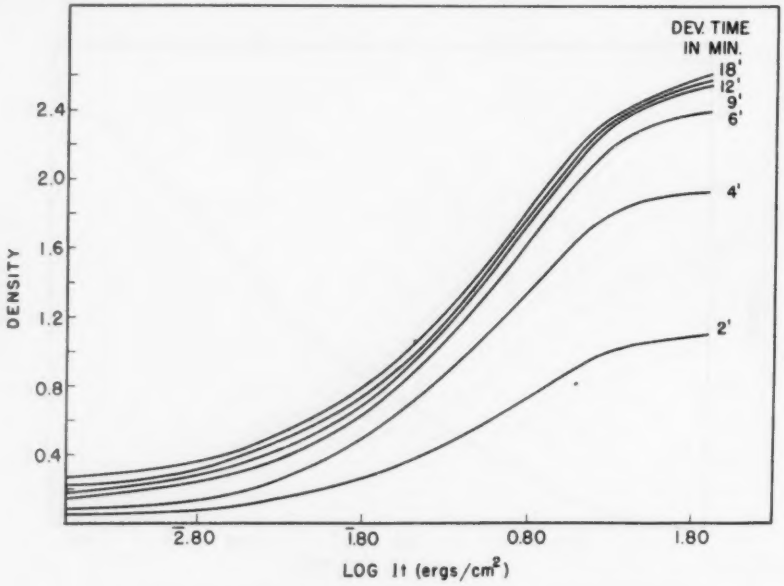
$$D(y) = 0.66F(y) + 1.80E\left(\frac{y}{m}\right), \qquad (7)$$

in which

$$m = 132,$$

having the intrinsic sensitivities, e.g., as follows:

for the 2-quanta curve: $\epsilon_2 = 201 \cdot 10^{-4}$; $1/2\epsilon_2 = 100 \cdot 10^{-4}$,
for the 1-quantum curve: $\epsilon_1 = 1.52 \cdot 10^{-4}$.



THE SAME EMULSION AS IN FIG. 14 DEVELOPED WITH FeOx

FIG. 16.

The calculated density values were plotted in Fig. 15 as circles to show the agreement between observed and calculated values. With the same exposures and with development in ferrous oxalate from 2 to 18 minutes at 20° C., this same film gives the quite normally shaped characteristic curves of Fig. 16. An analysis of the curve for this emulsion with 9 minutes' development, however, shows the same complexity of two sensitivity distributions among the grains, a two-quanta and a one-quantum curve, following the equation:

$$D(y) = 0.32F(y) + 2.20E\left(\frac{y}{m}\right), \qquad (8)$$

[17] Exposures were made with monochromatic light of $\lambda = 426$ mμ.

in which

$$m = 1622,$$

having the intrinsic sensitivities, ϵ, as follows:

for the 2-quanta curve: $\epsilon_2 = 450.8 \cdot 10^{-4}$; $1/2\epsilon_2 = 225 \cdot 10^{-4}$,
for the 1-quantum curve: $\epsilon_1 = 2780 \cdot 10^{-4}$.

It should be noted that the order of the two components is altered by changing the developer used, resulting in the two different shapes of the characteristic curves.

(*b*) At the beginning of these investigations, another break in the smoothness of the characteristic curves was observed. At that time the sensitometric limits of exposure were not yet so adjustable that reliable

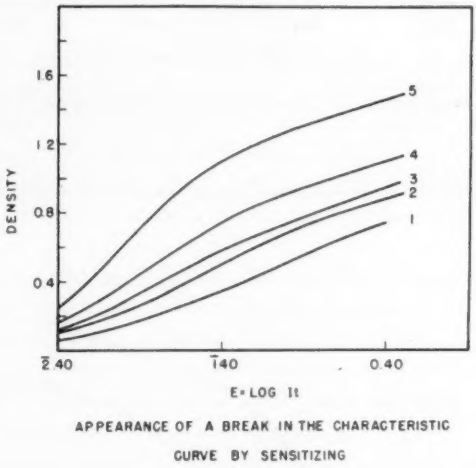
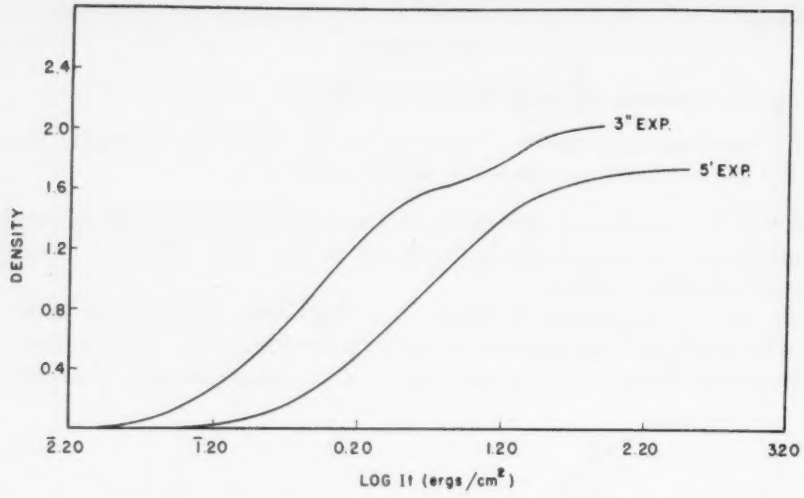


FIG. 17.

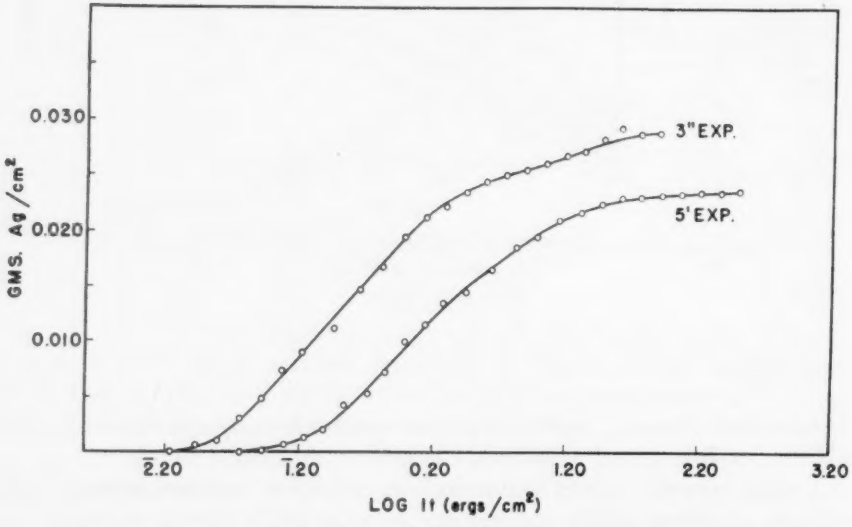
characteristic curves could be obtained for the range from $D = 0$ to $D_{max.}$. The sensitometric methods were limited to only a part of the characteristic curve. Such curves are useless for any quantum analysis. Only the experimental results are therefore described here.

A nonconverted silver bromoiodide emulsion containing 0.77 moles per cent. of silver iodide having an average grain size $\bar{a} = 2.93\mu^2$, and a dispersion of grain sizes $\sigma = 2.13\mu^2$, which is relatively so low that the uniformity in grain size $\bar{a}/\sigma = 1.37$ can be considered higher than the average, was sensitized with a suitable gelatin. The unsensitized emulsion with DK–50 development for 5 minutes at 20° C. gave Curve 1 of Fig. 17. Curves 2, 3, 4, and 5 were obtained during the progress of sensitizing. As can be seen, sensitizing produces a break in the characteristic curve, due probably to two different sensitivity distributions among the grains.[16]



ANOMALY PRODUCED BY A HIGH-INTENSITY EXPOSURE IN THE D/LOG It CURVE

FIG. 18.



ANOMALY IN THE Ag/LOG It CURVE PRODUCED BY A HIGH-INTENSITY EXPOSURE

FIG. 19.

(*c*) An unexplained effect was obtained with a silver bromoiodide emulsion containing 2.56 moles per cent. of silver iodide having an average grain size $\bar{a} = 0.50\mu^2$, coated on film which was exposed to two different intensity scales in such a way that one set of exposures was made in 3 seconds and the other in 5 minutes and that every exposure

step received the same radiant energy. Two different characteristic curves were obtained because of a reciprocity-law failure. These curves are shown in Fig. 18. Development was carried out in DK–50 developer for 5 minutes at 20° C.

The curve for the higher intensity shows an anomaly in the shoulder of the curve, which does not occur in the curve obtained with lower intensity. Mr. B. Hutchins, previously mentioned, obtained the silver equivalents of the densities, which are plotted as Ag/log *It* curves in Fig. 19. The same anomaly is detectable in these curves showing that the indentation in the shoulder of the high-intensity curve is not caused by an irregularity in the silver equivalent. It must, therefore, be a phenomenon produced in the development of the latent image.

This effect has been obtained repeatedly with the same emulsion.

SUMMARY.

1. From the disappearance of solarization on prolonged development, it can be assumed that there are on the grain development centers of different "sizes," the "smaller" of which take a longer time to start reduction of the grains than the "larger" ones.

2. It was shown anew that within certain limits the number of developed grains is proportional to the density for different development time.

3. Sensitizing increases the number of developed grains.

4. During development, the larger grains of an emulsion show greater developability than the smaller ones.

5. The effect of sensitizing diminishes with prolonged exposure in most cases. In a few cases in which there is little sensitizing effect, the sensitivity increases to a maximum and then diminishes. This increase indicates an initial activating effect during exposure.

6. There are indications that there exist different kinds of sensitivity specks, each of which has quantitatively a different sensitizing effect. This is seen by comparing the characteristic curve of an unsensitized with that of a sensitized emulsion.

7. A few anomalies are described, some of which can be explained and some cannot.

ROCHESTER, NEW YORK,
April 5, 1945.

New Uses For Fiberglas Mat.—Thin, porous mats of bonded glass fibers, originally developed for use as retainer mats in storage batteries and widely employed for this purpose, have now been successfully applied to a number of entirely new uses by the process and electrical industries.

Underground Pipe Wrap.

In roll form, the Fiberglas mat is being employed as a material for wrapping underground oil, gas and other pipe lines to protect them against corrosion and electrolytic action. The material can be wrapped around bitumen or coal tar-coated pipe, thus forming a continuous water-tight bond.

The glass fibers are non-corrosive to metals. The material has negligible moisture pick-up. Its tensile strength is preserved through a wide range of temperatures and exposure to organic solvents and soil acids. The millions of fine intertwined glass fibers reinforce a large amount of bitumen or coal tar coating for a given thickness of wrapping, as compared to other carriers.

Low-Loss Laminate.

Primarily because of the high tensile strength and non-hygroscopic characteristics of the individual glass fibers, the Fiberglas mats are being employed as the base for a new plastic laminated material. Possessing a low and stable loss factor over a wide frequency range, the laminate greatly extends the field for plastic coil forms, condenser spacers, stand-off insulators, etc., in radio, television and other high-frequency electronic devices.

In building up the laminate, the Fiberglas mats are impregnated with a thermo-setting aniline-formaldehyde resin, and are cured under high pressure. In addition to its low loss factor, the laminate possesses high strength, high temperature resistance, dimensional stability and resistance to fungus attack. Good machinability is another important property of the laminate.

Gaskets and Sheet Packing.

Fiberglas mat is also being employed as a base material for gaskets and sheet packing. The mat acts as a carrying medium for synthetic resins suitable for applications requiring resistance to heat, oil and acids. Glass-base gaskets now being manufactured show high pressure resistance, good chemical durability and little flow under flange pressure.

Made by Owens-Corning Fiberglas Corporation, Toledo, Ohio, Fiberglas mat is composed of fine glass fibers intertwined in random orientation and bonded together to form a thin, highly porous, felt-like material. The glass fibers have an average diameter of 5 ten-thousandths (0.0005) of an inch. Several types of binders—starch, gelatine, furfural or phenolic—may be used, depending upon the requirements of the end use.

Mats are produced in thicknesses ranging from 10 one-thousandths (0.010) to 50 one-thousandths (0.050) of an inch, and in standard widths of 22 and 36 inches in roll lengths of 150 and 300 feet.

R. H. O.

ANNUAL REPORT * OF THE BIOCHEMICAL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE (1945).†

BY

ELLICE McDONALD, M.D.,

Director.

RADIOACTIVITY AND BIOCHEMISTRY.

Radiation is the essence of life, although when we speak of it, we think of radiation as adapted by man: but, after all, without the sun and its radiations the world would not exist. The most profound problem is how the energetic radiations of the sun enable plants to produce their chlorophyll, for this substance is the means and mechanism of all life. It is the source of all the multitude of organic substances which provide the building blocks and pabulum of life in its many and varied forms.

However, as experimentalists we have mostly to do with radiations as adapted by man. This is really a thing of yesterday as the fifty year anniversary of the discovery of X-rays has only passed a short while ago. From what a discovery has so much development and information come? It seems to me that the two most enlightening discoveries of recent times have been that of X-rays by Roentgen and the synthesis of urea, the first organic substance ever artificially produced, by Woehler (1828). From one has come the whole range of discovery in physics culminating in the release of atomic energy and from the other has come the great science of organic chemistry and the multitude of substances with so much useful purpose in all forms of life.

It is beyond us at the present time to unfold the complicated and intricate mechanisms whereby the sun activates his materials for the production of so many substances for the metabolism and maintenance of life, but we can make some study of the effects of radiations and radioactive substances on living material whether that material is in the complete living state or whether it is broken down into various components, fractions or extracts which are within our range of possible study, for we are limited in our capacity by several things.

* The Biochemical Research Foundation has been working for two years with the Manhattan Engineering District upon biological aspects of the Atomic Bomb. We are allowed to say now that we have been doing this work and are assured that secrecy in regard to the biological aspects will be shortly removed. Such reports as detailed here comprises work that has been done in our laboratories outside of our efforts for atomic bomb research.

† Presented at Stated Meeting held on November 21, 1945.

Unlike the units of chemistry and physics, living cells are dynamic systems. They keep on living; if there is too much experimental injury to them, they die. And they are different after they die than when they were living. They won't stay put. So that the biochemist must shoot them on the fly and try to determine what changes are produced by his experimental methods and with the few criteria he has at hand. Then again the cell, the unit of life, does not consist of a cell alone: it exists in an environment and the cell and its environment are one and, if the cell is removed from its environment, it is no more a cell than the dead salmon is the fish which lived and lunged and played from the sparkling fresh streams to the briny reaches of the salt sea.

Not only has the cell its environment but it is a complicated system which consists at least of (1) the nucleus of the cell, (2) the cell protoplasm, (3) the cell boundary and (4) the environment. It is in the environment that most biochemical observations are done because we can remove portions of it without killing the cell. This is only to simplify the matter; the whole system is colloidal in character and no doubt there are colloidal systems within colloidal systems and all these have the embarrassing quality of acting very like chemical systems of true solution character. There are multitudes of catalysts, almost all of them enzymes, which determine the scope and character of all those chemical reactions that we call life.

For life is a chemical phenomenon, that is so axiomatic that not even the physicists whose protestations and prophecies are reaching higher than the atomic bomb, can say that life is a phenomenon in physics. We are relieved that one phase of life is left to our research in mundane affairs.

Still in experiments into living processes, the complicated vital system must be considered and, when radiation is studied, the explanation as to its processes and changes in the dynamic living system cannot be those based upon changes produced in the bombardment of a pure chemically stable substance by neutrons or other types of radiation. We are also entitled to ask where does the radiation produce its changes: in what component of the cell system, the nucleus, the protoplasm, the cell boundary or the environment. And what is the mechanism of the reactions? Cause and end result give little information. If enough radiation of the proper kind is delivered the cell, the animal will die; we can kill the animal as easily with a hammer and with much less elaborate apparatus and trouble. Death tells us little: it is with life we have to do. Life goes on and we are interested in how it goes on, so that when we consider the effects of radiation and radioactive materials we are interested in the *mechanism*. Such an explanation of radiation effects as being various degrees of ionization leaves us cold: that may satisfy for physical systems, but we may ask Ionization where, how and to what substances?

The interest of the biochemist is not in the effects and mechanism of radiation alone: there are much wider ramifications. Radiation as adapted by man is a destructive force. We are interested in its control and mechanism, for the explanation of the mechanism may aid us to the understanding of many other processes of life. An outstanding example is that of cancer which was found to be influenced by X-rays and even in some cases cured if enough radiation was concentrated at the proper spot. This led to bigger doses of X-rays and more severe kinds of radiation, such as the gamma rays of radium. This was based upon the old myth that "if some is good, more is better" which is neither true in life nor medicine.

The trouble is that radiations of this character are destructive forces and they can kill all cells whether cancerous, diseased or normal. They shoot into the flock and do not single out the required bird. If they could be concentrated on the diseased cell then it would be ideal, but, with such a destructive force and the complicated vital system, it is difficult to imagine control so that special cells alone are destroyed. In some cases, small normal portions of the body may be sacrificed, so that the diseased one may be killed as in the treatment of external cancers by radium. This is analogous to the surgical excision of a cancer which cannot be done without sacrificing healthy tissue surrounding it.

We have two ways of making use of radiation in disease: one to reduce the destructive power so that weaker cells which may be diseased are alone killed and the other to discover the mechanisms of radiation, to find out how the vital system is altered so as to produce the killing effects and to produce these within the body where they can be controlled and localized.

This is our interest and it is in cancer where we have worked so long. Although a great part of our laboratory is now working on radioactive problems, we are really all working on cancer. For if we can explain the mechanism of radiation we can apply it in less destructive forms to the treatment of cancer and other forms of disease as well as going a step forward in explaining many vital processes. We are like a consulting geologist hired to survey a mine. To the disgust of the miners, the last place he wants to go is down the shaft. He wants to explore the surrounding geological formations before he will venture to give an opinion on the ore body. So that a visitor in our laboratories might think that we were working very little upon the cancer problem while actually, although concerned very considerably with radioactive problems, we were never more occupied with the study of cancer.

In radiations in general from the biochemical point of view, we have the following divisions of the spheres of study.

1. The use of radioactive materials as biological tracers.

a. To put radioactive materials in such form as they normally exist into animal bodies and study where they go, by means of Geiger counters, in what proportions they exist and at the same time try to determine what changes they produce. This we have done with many substances, such as radioactive iodine, radioactive iron, radioactive sodium phosphate, etc., as phosphate is a normal constituent, and with several substances which are not normal to the animal body, such as radioactive gold. These latter were studied from the point of view of their effect on cancer growth. We can in this way study the behavior of minute amounts of radioactive substances as the extreme sensitivity of electrometric methods makes it possible to detect these radioactive substances in much smaller concentrations than can be done by chemical methods.

b. The production in radioactive form of complex organic substances which normally exist in the animal body and the study of their distribution and fate. An example of this is glutathione, a normal and important catalyst in the body which was produced by growing yeast on a medium containing radioactive sulfate and extracting the gultathione from that yeast: the resulting glutathione was radioactive.

Another example was obtaining radioactive bacteria by cultivation on a medium containing radioactive materials. The resulting bacteria could be traced as to their distribution, fate and amounts, by means of Geiger counters. Some bacteria retained such radioactivity that there was sufficient to blacken a photographic plate by that means alone, so that these radioactive bacteria emitted sufficient energy to take a picture without any other source of light. Radioactive corn roots and a number of other radioactive appurtenances to life have also been produced.

c. The production of radioactive drugs wth the aim of finding out where they go and what is their action. It's an old medical saying that "A drug is put into the mouth and after that it is in God's hands." Little is known of the mechanism of drug action although the light is beginning to gleam through the darkness. Sulfanilamide, for example, is an important new and life-saving drug of which we would like to know the mechanism of action. We would like to know where it goes and perhaps from that move a step along towards discovering the mechanism of its action. We have succeeded in synthesizing two radioactive compounds which are intermediates in the synthesis of the important drug sulfanilamide.

An enormous area of research in this field is open in the study of the vitamins and hormones, so many of which are now produced synthetically. This no doubt has been halted by the war effort which is acclaimed as producing great advances in research while really it did little but elaborate and enlarge previous discoveries and has really hindred a great mass of research which would have been done had not

the scientists been drawn away into the war efforts which were necessary but no one need applaud this as advance in the character and amount of research.

2. The use of radioactive materials in physicochemical studies. When a substance containing impurities has been rendered radioactive by bombardment with charged particles of high energy or neutrons, minute traces of elements as impurities may be detected because of their characteristic half-lives. With the help of chemical separation after bombardment, impurities present to the order of one part in a million can sometimes be detected. Another example of their use is in the determination of the solubility of slightly soluble salts. With the aid of a radioactive indicator, the solubility of such salts may be measured with no more difficulty than that involved in simple weighing. The use of radioactive elements in connection with measurements of the thickness of the adsorbed layer on powdered adsorbents is another instance, as is also the investigation of the distribution of metallic elements in alloys. These radioactive materials have also been used in the confirmation of reactions on which previous rate and equilibrium measurements have been made by customary methods.

3. The treatment of disease by means of radioactive substances. The fatal disease of the blood, leukemia, is one example of this. In chronic cases we have had good success in ameliorating the disease by treatment with radioactive phosphorus (P^{32}) in the form of sodium phosphate intravenously injected at the pH of the blood plasma. Some of these chronic cases can be kept within normal range for at least two years. After being brought to approximately normal in their blood counts, a periodic dose will usually hold them there. The acute cases of leukemia, however, are so devastatingly rapid that they cannot be helped. Polycythemia, or too many red blood cells, also responds readily to treatment by radioactive substances.

In our study of the effects of short ultraviolet wavelengths on cells we found that ultraviolet light of the wavelength of 2536 Ångström units was extremely lethal and probably the most lethal radiation yet discovered, killing cells within a second or so, when these same cells required 20,000 Roentgen units for 24 hours for lethal effects. The result was apparently due to absorption of the quanta by the nucleic acid of the cell nucleus. We hoped to use this wavelength on cancer, but ultraviolet light will penetrate only a millimeter or so, and the tissues themselves prevent the penetration.

The hope was to bring this wavelength into direct proximity to the cell itself, so that the effect might be applicable. We considered the production of fluorescent substances which in solution might be injected into a cancer and then activated from outside by X-rays so that the fluorescent substance in proximity to the cancer cells would give off the required wavelength of 2536 Ångström units.

We studied several thousand substances to determine their fluorescence in solution, but were unable to get any that fluoresced lower than 2600 Ångström units in solution. Apparently instead of obtaining wavelengths shorter than that the substance breaks down. In the proteins, this is apparently at the polypeptide linkages: at least there is some evidence that this is so. This project if it had worked would have been ideal, but perhaps the spade work we did may sometime in the future bear fruit.

4. The mechanism of radiation is for us the most important problem, for if the mechanism is even partially understood it may be possible to produce similar results with less destructive means. If radiations can sometimes cure cancer it may be possible to copy the means of the action on the cells without its general destructive action. This is quite within the bounds of possibility.

We have found something about the lethal mechanism of the shorter wavelengths of ultraviolet as due to the absorption of such a line as 2536 Ångström units in the nucleus of the cell and the nucleic acid there. It is an entirely unwarranted assumption that, if one type of radiation has a certain mechanism in destroying cells, that another type of radiation has a similar mechanism only differing in the amount of energy applied. Certainly this is not the case in the lethal effects of shorter ultraviolet and X-rays. There is no reason to suppose, certainly no proof, that the mechanism of the destructive effects of high voltage X-rays and fast or slow neutrons is similar in character, even though the few criteria we have are somewhat similar. Causes are somewhat similar and end result criteria may be alike, but the wide gap of intermediate mechanism may be quite different. There are so many things in the complicated structure of the cell system and its environment that a premise of similar mechanism is unwarranted. For example, one may knock out the catalyst of a necessary enzyme, one may release an inhibitor of another enzyme, either of which would be sufficient for killing of the cell.

Mechanism of radiation is therefore our most important problem and that at present is our main effort.

One prominent feature in both X-ray and slow neutron radiation is the rapidity with which white blood corpuscles are removed from the blood stream If the dose is sufficient, all white blood corpuscles disappear: there will not be one remaining in the blood sample nor in the bone marrow. When it is considered that these white blood corpuscles are renewed several times daily, probably four or five times, and that fresh white blood corpuscles introduced into the blood stream are not killed by these radiations, it is obvious that something has happened to prevent their reproduction and their restoration into the blood stream. They are not killed as such: they live their short span and are not renewed. It is as if a whole community were suddenly

sterilized, lived their lives out and there were not any more. There are other changes in other cells, but this complete elimination of white blood corpuscles with large doses of radiation is one of the most salient facts.

Apparently one of the chief effects of radiation is a disturbance of the metabolism of the nucleic acid of the cell. One striking effect is the significant increase in ultraviolet absorption after X-ray and gamma radiation. Another is the accumulation of the pentose nucleotides, the mononucleotides within the cell after radiation. This means that the nucleic acid, most probably ribose nucleic acid, has been broken down into smaller fractions with a much smaller molecular weight.

These mononucleotides have the capacity of inhibiting certain enzymes associated with the redox equilibria and the release of these within the cell can regulate its metabolic activities and most probably produce a variation in the redox equilibria within the nucleus which is so associated with the division and the multiplication of cells as exists in cancer. The smaller molecular weight of the mononucleotides as compared with the great molecular weight of ribose nucleic acid may make it possible that there be penetration of the smaller molecule through the cell wall which could not be penetrated by the larger molecule nor the large molecular enzymes.

There is no doubt that prevention of cell division and the cure of cancer depends upon an alteration in the oxidation-reduction equilibrium necessary to cell growth as I stated fourteen years ago. The question is how to produce this alteration and it may be that the study of the mechanism of radiation will lead us to the answer of this question.

It is not a question of greater and more severe doses of radiation, million volt X-rays or 15 million volt cyclotrons. These can destroy all tissues with ease without selection and leave destruction in their wake. The problem is to find the disturbance in the metabolic functions of the cells which causes their death and to simulate these conditions in a most restrained and localized fashion and apply the methods upon those sections of cell system (which includes the environment) as are concerned with the division and multiplication of cells, significant of cancer. If, in cancer, one new cell nucleus can be formed every 27–30 hours, it is obvious that prevention of the continued formation of these new cells will prevent the growth of cancer.

So our study of the mechanism of radiation is a study of the metabolism of the cell as affected by radiation and since the metabolism of the cell is bound up in its enzyme systems these latter are a chief point of attack.

If it is possible by chemical means to inhibit the enzymes concerned in cell metabolism, which may be the secondary effect of radiation upon cancerous cells, there will be a good hope of restraining cell growth.

Motor Fuels From Farm Wastes.—A process for the chemical conversion of such agricultural residues as corncobs, sugarcane bagasse, peanut shells, flax shives, oat hulls and cotton seed hulls and burrs into liquid motor fuel and other commercially valuable products is moving into the semi-works stage of experimentation at the Department of Agriculture's Northern Regional Research Laboratory at Peoria, Illinois.

Results of experimental laboratory investigations by Department chemists indicate that from 90 to 95 gallons of liquid motor fuel can be obtained from a ton of corncobs or cottonseed hulls and that about half of this is in the form of ethyl alcohol. A continuous process of the treatment (or saccharification) of corncobs and cottonseed hulls has been worked out by Drs. E. C. Lathrop and J. W. Dunning of the Peoria Laboratory and gave such promising results in the laboratory stage that the work is to be enlarged so that it may now be evaluated on a semi-commercial scale. This enlarged program calls for a new building with modern equipment and a staff of approximately 15 of the best scientists obtainable.

Around 200 million tons of farm wastes are produced each year. Much of this is plowed back into the soil to help maintain fertility and prevent erosion. It is estimated that something like 100 million tons might be available for use in this new process for the production of liquid motor fuels. Corncobs will be the first of these so-called waste materials to be tried on a large scale in the new semi-works plant. Research on the other residues will be started later as the work progresses.

The ground residue or raw material is first treated with an acid, or saccharified, to produce concentrated solutions of dextrose and xylose, which are fermented into alcohol, butanol, and acetone. Under the new saccharification process relatively pure, separate solutions of 5-carbon sugars and 6-carbon sugars are obtained. These solutions contain from 10 to 15 per cent. of sugar, which is about the content that is used in industrial fermentation. These are the materials from which liquid motor fuels may be made. Furfural and lignin are at present obtained as byproducts of the process. The new plant is designed to produce approximately 2,000 pounds of dextrose, 1,600 pounds of xylose, 1,000 pounds of lignin, and 200 pounds of furfural from 6,600 pounds of raw residue in an eight-hour operating period.

This particular study on the industrial utilization of farm waste material is a part of a broad program authorized by Congress in 1944 for large-scale research on the production of synthetic liquid fuels from coal, oil shale, agricultural and forestry products, and other substances. Work on coal and oil shale will be carried on by the Bureau of Mines of the Department of the Interior. Investigations on agricultural and forestry products will be handled by the Department of Agriculture.

R. H. O.

ON THE METHOD OF VAN DER POL AND ITS APPLICATION TO NON-LINEAR CONTROL PROBLEMS.

BY

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The purpose of this paper is the construction of approximate differential equations (Formulae 32 below) of a mechanical or electrical pseudo-linear oscillatory system with many degrees of freedom. These equations must be applicable to the study of the transitory or quasi-periodic processes and not only of the periodic vibrations as in the previous communication of the author [1] (the figures in brackets refer to the list of literature at the end of the paper). The important case of an automatically controlled system is chosen as an example and made the object of a special investigation.

The reasoning is based essentially on the ideas of E. V. Appleton and B. van der Pol [2, 3] and their interpretation by L. Mandelstam and N. Papalexi [4]. This method has been applied by its originators and other authors to many problems. N. Kryloff and N. Bogoliùboff [5] considered a system characterized, after reduction, by an equation of any finite order with a non-linear term depending on one of the coordinates only. The study of a still more general system is attempted in the following. For the attainment of the aspired result it is necessary to begin with an extension of the classical transformation to normal coordinates (Formulae 29 below) to the sets of linear equations with constant coefficients of the most general type of E. J. Routh and E. L. Ince.

1. *Preliminary Consideration of Linear Systems with Constant Coefficients and their Normal Coordinates.* Consider a set of linear equations

$$\sum_{k=1}^{n} f_{jk}(D)x_k = \varphi_j(t)(j = 1, \cdots, n), \tag{1}$$

where the symbol

$$D = \frac{d}{dt} \tag{2}$$

is the differential operator and x_k denote the unknown generalized coordinates of the system depending on the time variable t; the f's and φ's are given functions of their arguments, the first set being simply polynomials in D of any finite degrees with constant coefficients. Let us suppose: (a) that the determinant $\Delta(D)$ of the operational matrix $f(D) = ||f_{jk}(D)||$ is not identically zero and (b) that all roots $\kappa_\nu(\nu = 1,$

$\cdots, N$) of the characteristic equation $\Delta(D) = 0$ are simple. Further assumptions will follow when proceeding to non-linear systems but they need not be made from the outset.

Introducing the column matrices

$$x = \left\| \begin{matrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{matrix} \right\|, \qquad \varphi(t) = \left\| \begin{matrix} \varphi_1(t) \\ \cdot \\ \cdot \\ \cdot \\ \varphi_n(t) \end{matrix} \right\|, \tag{3}$$

we express (1) as a single matric equation

$$f(D)x = \varphi(t); \tag{4}$$

the multiplication of matrices here and later on must be effected in conformance with the usual rules [6].

If $F_{jk}(D)$ denotes the co-factor of $f_{jk}(D)$ in $\Delta(D)$ then one at least of the quantities $F_{jk}(\kappa_\nu)$ must be other than zero; let

$$F_{l(\nu),\, m(\nu)}(\kappa_\nu) \neq 0. \tag{5}$$

Using the known relation

$$F_{l(\nu),\, m(\nu)}(\kappa_\nu) F_{kj}(\kappa_\nu) - F_{l(\nu),\, j}(\kappa_\nu) F_{k,\, m(\nu)}(\kappa_\nu) = 0,$$

we obtain

$$F_{kj}(\kappa_\nu) = X_j^{(\nu)} B_k^{(\nu)}, \tag{6}$$

wherein

$$X_j^{(\nu)} = S_\nu \frac{F_{l(\nu),\, j}(\kappa_\nu)}{F_{l(\nu),\, m(\nu)}(\kappa_\nu)}, \qquad B_k^{(\nu)} = \frac{1}{S_\nu} F_{k,\, m(\nu)}(\kappa_\nu) \tag{7}$$

and S_ν is some chosen factor of proportionality. If X_ν, B_ν are column and row matrices defined by

$$X_\nu = \left\| \begin{matrix} X_1^{(\nu)} \\ \cdot \\ \cdot \\ \cdot \\ X_n^{(\nu)} \end{matrix} \right\|, \qquad B_\nu = \| B_1^{(\nu)}, \cdots, B_n^{(\nu)} \| \tag{8}$$

and if

$$F(D) = \| F_{kj}(D) \| \tag{9}$$

is the adjoint matrix of $f(D)$, that is the transposed of the matrix of the co-factors $F_{jk}(D)$, then we can express (6) as a matric relation

$$F(\kappa_\nu) = X_\nu B_\nu \tag{10}$$

required later.

The elements of X_ν being proportional to the co-factors of the elements of the $l(\nu)$-th row in $\Delta(\kappa_\nu)$ the column $X_\nu \exp \kappa_\nu t$ is a particular

solution of the homogenous system $f(D)x = 0$. The sum

$$\sum_{\nu} X_{\nu} G_{\nu} \exp \kappa_{\nu} t$$

with arbitrary constants G_{ν} represents the general solution.

By virtue of the above assumption (a) the matrix $f(D)$ is non-singular and we can introduce the inverse $f^{-1}(D)$ defined as usual [6]. The particular solution of the non-homogenous system can then be writtn in the operational form

$$f^{-1}(D)\varphi(t) = \frac{F(D)}{\Delta(D)}\varphi(t) = S(D)\varphi(t) + \sum_{\nu} \frac{H_{\nu}}{D - \kappa_{\nu}}\varphi(t)$$

with

$$S(D) = \text{integral part of } F(D)/\Delta(D), \tag{11}$$

$$H_{\nu} = \left[\frac{F(D)(D - \kappa_{\nu})}{\Delta(D)}\right]_{D=\kappa_{\nu}} = \frac{F(\kappa_{\nu})}{\Delta'(\kappa_{\nu})}.$$

The interpretation of the terms containing partial fractions is

$$\frac{1}{D - \kappa_{\nu}}\varphi(t) = \exp \kappa_{\nu} t \int_0^t \varphi(\tau) \exp(-\kappa_{\nu}\tau) d\tau,$$

where τ is the variable of integration.

With this the general solution of the non-homogenous system can be written as

$$x = \sum_{\nu} X_{\nu} G_{\nu} \exp \kappa_{\nu} t + S(D)\varphi(t) + \int_0^t \beta(t - \tau)\varphi(\tau) d\tau,$$

where

$$\beta(t) = \sum_{\nu} \frac{F(\kappa_{\nu})}{\Delta'(\kappa_{\nu})} \exp \kappa_{\nu} t. \tag{12}$$

In order to express the solution in real form we shall from now on use the notation $\kappa_g (g = 1, \cdots, N')$ for the real roots only and introduce special symbols $\epsilon_h \pm i\omega_h$ $(h = N' + 1, \cdots, N' + N'')$ for the pairs of complex ones. Next we shall require, besides x, the expressions of its derivatives x^{ν} with respect to t. Denoting by β^{ν} the derivatives of β and by C_h the column matrices analogous to X_g but corresponding to complex roots we obtain

$$x^{\nu} = \sum_{g} X_g \kappa_g^{\nu} G_g \exp \kappa_g t$$
$$+ \sum_{h} [C_h(\epsilon_h + i\omega)^{\nu} G_h \exp(\epsilon_h + i\omega_h)t + \bar{C}_h(\epsilon_h - i\omega_h)^{\nu} \bar{G}_h \exp(\epsilon_h - i\omega_h)t]$$
$$+ S^{(\nu)}(D)\varphi(t) + \int_0^t \beta^{\nu}(t - \tau)\varphi(\tau) d\tau;$$

the bars over the letters are employed as usual to indicate the conjugate

imaginaries and the meaning of the symbols $S^{(\nu)}(D)$ is

$$S^{(\nu)}(D) = D^\nu S(D) + \beta(0)D^{\nu-1} + \dot{\beta}(0)D^{\nu-2} + \cdots + \beta^{\nu-1}(0). \quad (13)$$

Alternatively

$$x^\nu = \sum_g X_g \kappa_g{}^\nu G_g \exp \kappa_g t + 2\sum_h \exp \epsilon_h t Re[C_h(\epsilon_h + i\omega_h)^\nu G_h \exp i\omega_h t]$$
$$+ S^{(\nu)}(D)\varphi(t) + \int_0^t \beta^\nu(t-\tau)\varphi(\tau)d\tau.$$

Using (10), (12) we have also the expressions

$$\int_0^t \beta^\nu(t-\tau)\varphi(\tau)d\tau = \sum_g X_g \kappa_g{}^\nu K_g \exp \kappa_g t$$
$$+ 2\sum_h \exp \epsilon_h t Re[C_h(\epsilon_h + i\omega_h)^\nu K_h \exp i\omega_h t] \quad (14)$$

with

$$K_g = \frac{1}{\Delta'(\kappa_g)} B_g \int_0^t \varphi(\tau) \exp(-\kappa_g \tau)d\tau,$$
$$K_h = \frac{1}{\Delta'(\epsilon_h + i\omega_h)} B_h \int_0^t \varphi(\tau) \exp(-\epsilon_h - i\omega_h)\tau d\tau.$$

Each of the matrices K_g, K_h being a product of the row B_g or B_h with a column matrix contains only one row and one column; they are thus scalars, namely

$$K_g = \frac{1}{\Delta'(\kappa_g)} \sum_{k=1}^{n} B_k{}^{(g)} \int_0^t \varphi_k(\tau) \exp(-\kappa_g \tau)d\tau,$$
$$K_h = \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \sum_{k=1}^{n} B_k{}^{(h)} \int_0^t \varphi_k(\tau) \exp(-\epsilon_h - i\omega_h)\tau d\tau. \quad (15)$$

Using (14) and putting

$$\epsilon_h + i\omega_h = c_h \exp i\zeta_h, \quad (16)$$

$$G_g + K_g = A_g, \qquad G_h + K_h = \tfrac{1}{2}A_h \exp i\theta_h, \quad (17)$$

we bring the expressions of x^ν into the form

$$x^\nu = S^{(\nu)}(D)\varphi(t) + \sum_g X_g \kappa_g{}^\nu A_g \exp \kappa_g t$$
$$+ \sum_h \exp \epsilon_h t Re[C_h c_h{}^\nu A_h \exp i(\omega_h t + \theta_h + \nu\zeta_h)]. \quad (18)$$

Using as before the notations $X_j{}^{(g)}$ and the analogous notations $C_j{}^{(h)}$ for the elements of the columns C_h we set

$$X_j{}^{(g)}\kappa_g{}^\nu = X_{j\nu}{}^{(g)}, \qquad C_j{}^{(h)} = N_j{}^{(h)} \exp i\gamma_j{}^{(h)}, \qquad N_j{}^{(h)}c_h{}^\nu = N_{j\nu}{}^{(h)} \quad (19)$$

and obtain the following expressions for the elements of the columns x^v

$$x_j^v = \sum_{k=1}^{n} S_{jk}^{(v)}(D)\varphi_k(t) + \sum_g X_{jv}^{(g)} A_g \exp \kappa_g t + \sum_h N_{jv}^{(h)} A_h \exp \epsilon_h t \cos (\omega_h t + \theta_h + \gamma_j^{(h)} + v\zeta_h), \quad (20)$$

where $S_{jk}^{(v)}(D)$ are the elements of the matrices $S^{(v)}(D)$. If m_j denotes the order of the highest derivative $D^{m_j}x_j$ of x_j occurring in the equations then it is sufficient to give to v for each j the values

$$v = 0, 1, \cdots, m_j', \quad (21)$$

the number m_j' being equal to the greater of the integers 0 and $m_j - 1$ or to both if $m_j = 1$. In writing the formulae corresponding to $v = 0$ we assume the convention $x_j^0 = x_j$, $S_{jk}^{(0)}(D) = S_{jk}(D)$ and so obtain the expressions of the unknowns x_j themselves.

In view of the subsequent application to non-linear systems we differentiate the relations (17) and substitute for K_g, K_h from (15). The resulting equations

$$\frac{dA_g}{dt} = \frac{\exp(-\kappa_g t)}{\Delta'(\kappa_g)} \sum_{k=1}^{n} B_k^{(g)} \varphi_k(t) \quad (g = 1, \cdots, N'), \quad (22)$$

$$\frac{dA_h}{dt} + iA_h \frac{d\theta_h}{dt} = \frac{2 \exp[-\epsilon_h t - i(\omega_h t + \theta_h)]}{\Delta'(\epsilon_h + i\omega_h)} \times \sum_{k=1}^{n} B_k^{(h)} \varphi_k(t) \quad (h = N' + 1, \cdots, N' + N'') \quad (23)$$

may be used as well to determine the $N' + 2N''$ quantities A_g, A_h and θ_h as functions of t.

For a homogenous system ($\varphi_1 \equiv \cdots \equiv \varphi_h \equiv 0$) the formulae (20) take the form

$$x_j^v = \sum_g X_{jv}^{(g)} A_g \exp \kappa_g t + \sum_h N_{jv}^{(h)} A_h \exp \epsilon_h t \cos (\omega_h t + \theta_h + \gamma_j^{(h)} + v\zeta_h) \quad (v = 0, 1, \cdots, m_j' \quad \text{and} \quad j = 1, \cdots, n), \quad (24)$$

the quantities A_g, A_h, θ_h being constant. We have thus in (22), (23) the equations of the variation of constants for a linear system with constant coefficients of the most general type (1).

Instead of the supposition (a) formulated at the beginning of this section we shall from now on assume (c) that the determinant of the coefficients of the highest derivatives and all integers m_j are other than zero. Then $m_j' = m_j - 1$ for each j. The term of the highest degree

$$u = \sum_{j=1}^{n} m_j \quad (25)$$

will be present in the development of $\Delta(D)$ so that $u = N' + 2N''$ and the degree of $F(D)$ in D will be lower than that of $\Delta(D)$. It follows that the integral part $S(D)$ of $F(D)/\Delta(D)$ vanishes and those of the expressions (24) which correspond to $v = 0$ and determine the unknowns x_j themselves hold for the non-homogeneous system though then the A's and the θ's are no longer constants.

Moreover, it is readily shown that this is also the case with all other expressions of the set. To verify this introduce for each k the derivatives $x_k^{m_k-1}, \cdots, \dot{x}_k$ as additional unknowns and augment the system (1) by trivial relations

$$\frac{dx_k^{m_k-2}}{dt} - x_k^{m_k-1} = 0, \cdots, \frac{d\dot{x}_k}{dt} - \ddot{x}_k = 0, \qquad \frac{dx_k}{dt} - \dot{x}_k = 0$$

$$(k = 1, \cdots, n).$$

An enlarged system of u equations of the first order with u unknowns is thus formally obtained. The expressions (24) with constant A's and θ's determine the unknowns of this enlarged system for the case when all φ's are zero. According to the above remark applicable to any set of equations satisfying the conditions (b), (c), all expressions considered must hold for the non-homogeneous equations so that the polynomials $S_{jk}^{(v)}(D)$ vanish identically for $v = 0, 1, \cdots, m_j - 1$ and $j, k = 1, \cdots, n$; which was to be demonstrated.

The relations (24) may be regarded as the equations of transformation expressing u functions $x_j^v (v = 0, 1, \cdots, m_j - 1$ and $j = 1, \cdots, n)$ in terms of $u = N' + 2N''$ new unknowns A_g, A_h, θ_h. These resemble the normal coordinates in that the equation (22) for each A_g is independent of the others, as well as the two equations satisfied by each pair $A_h\theta_h$ and obtained from (23) by separation into real and imaginary parts. A still closer generalization of the classical linear transformation to normal coordinates can be obtained using the variables $\xi_g = A_g \exp \kappa_g t$, $\xi_h = A_h \exp \epsilon_h t \cos(\omega_h t + \theta_h)$, $\eta_h = -A_h \exp \epsilon_h t \sin(\omega_h t + \theta_h)$. But for this and other particulars and generalizations not strictly necessary in the following we refer to an article on normal coordinates that the author proposes to publish.

2. *Equations of a Non-Linear System Expressed in Normal Form.* We proceed to the set of equations

$$\sum_{k=1}^{n} f_{jk}(D)x_k = \varphi_j(x_1, \cdots, x_n, t) \quad (j = 1, \cdots, n), \tag{26}$$

where the f's are the same as before but the functions on the right, in general non-linear, now depend not only on t but also on the unknowns $x_1, \cdots, x_n$. These functions may also involve for some or for each k the derivatives $D^{m_k-1}x_k, \cdots, Dx_k$ which are not indicated explicitly merely to avoid a complication of notation.

We may still apply the transformation (24) but in the equations (22), (23) satisfied by the new variables A_g, A_h, θ_h we must replace $\varphi_k(t)$ by $\varphi_k(x_1, \cdots, x_n, t)$. We may also introduce two other sets of unknowns, the "amplitudes"

$$\xi_g = A_g \exp \kappa_g t, \qquad a_h = A_h \exp \epsilon_h t \tag{27}$$

and the angular variables

$$u_h = \omega_h t + \theta_h. \tag{28}$$

The transformation (24) then takes the form

$$\boxed{\begin{gathered} x_j^{\nu} = \sum_g X_{j\nu}^{(g)} \xi_g + \sum_h N_{j\nu}^{(h)} a_h \cos (u_h + \gamma_j^{(h)} + \nu\zeta_h) \\ (\nu = 0, 1, \cdots, m_j - 1 \qquad \text{and} \qquad j = 1, \cdots, n) \end{gathered}} \tag{29}$$

and we have

$$\frac{d\xi_g}{dt} = \left(\kappa_g A_g + \frac{dA_g}{dt} \right) \exp \kappa_g t = \kappa_g \xi_g + \frac{dA_g}{dt} \exp \kappa_g t$$

or, using (22) and replacing there $\varphi_k(t)$ by $\varphi_k(x_1, \cdots, x_n, t)$

$$\frac{d\xi_g}{dt} = \kappa_g \xi_g + \frac{1}{\Delta'(\kappa_g)} \sum_{k=1}^{n} B_k^{(g)} \varphi_k(x_1, \cdots, x_n, t) \qquad (g = 1, \cdots, N'). \tag{30}$$

Similarly

$$\begin{aligned} \frac{da_h}{dt} + ia_h \frac{du_h}{dt} &= \epsilon_h a_h + \frac{dA_h}{dt} \exp \epsilon_h t + ia_h \left(\omega_h + \frac{d\theta_h}{dt} \right) \\ &= (\epsilon_h + i\omega_h) a_h + \left(\frac{dA_h}{dt} + iA_h \frac{d\theta_h}{dt} \right) \exp \epsilon_h t \end{aligned}$$

and substituting from (23), replacing again $\varphi_k(t)$ by $\varphi_k(x_1, \cdots, x_n, t)$ and separating into real and imaginary parts,

$$\begin{gathered} \frac{da_h}{dt} = \epsilon_h a_h + 2 \operatorname{Re} \frac{\exp (-iu_h)}{\Delta'(\epsilon_h + i\omega_h)} \sum_{k=1}^{n} B_k^{(h)} \varphi_k(x_1, \cdots, x_n, t), \\ \frac{du_h}{dt} = \omega_h + \frac{2}{a_h} \operatorname{Im} \frac{\exp (-iu_h)}{\Delta'(\epsilon_h + i\omega_h)} \sum_{k=1}^{n} B_k^{(h)} \varphi_k(x_1, \cdots, x_n, t) \\ (h = N' + 1, \cdots, N' + N''). \end{gathered} \tag{31}$$

The x's and their derivatives on the right-hand sides of (30), (31) must be thought of as replaced by their expressions (29).

So far we have a mere change of the dependent variables so that (30), (31) are exact equations and their integration may present no less difficulties than that of the original set. But we can now treat the problem approximatively confining ourselves to the systems satisfying

the last four general assumptions that we shall make, viz. (d) that the frequencies ω_h are linearly independent in the sense that there exists no relation of the form

$$\sum_h g_h \omega_h = 0,$$

where the g's are integers not all zero, (e) that the linear terms $\kappa_g \xi_g$, $\epsilon_h a_h$ in (30) and in the first set (31) are small in comparison with $\omega_h a_h$, (f) that the non-linear corrections $\varphi_k(x_1, \cdots, x_n, t)$ are also small for all values of the arguments considered (pseudo-linear systems), and (g) that the variation of the φ's due to t if it actually figures as an argument is slow when compared to that caused by the u's. Then ". . . we may expect the possible building-up or decay of amplitudes to occur slowly compared with the oscillations themselves . . ." [3] and are justified in replacing the terms on the right-hand sides of (30), (31) which contain the φ's by their mean values with respect to the u's. That is, in virtue of (d),

$$\begin{aligned}
\frac{d\xi_g}{dt} &= \kappa_g \xi_g + \frac{1}{(2\pi)^{N''} \Delta'(\kappa_g)} \\
&\qquad \times \sum_{k=1}^{n} B_k^{(g)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k du_{N'+1} \cdots du_{N'+N''}, \\
\frac{da_h}{dt} &= \epsilon_h a_h + \frac{1}{2^{N''-1}\pi^{N''}} \operatorname{Re} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \\
&\qquad \times \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_{N'+1} \cdots du_{N'+N''}, \\
\frac{du_h}{dt} &= \omega_h + \frac{1}{2^{N''-1}\pi^{N''} a_h} \operatorname{Im} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \\
&\qquad \times \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_{N'+1} \cdots du_{N'+N''} \\
&\qquad (g = 1, \cdots, N' \quad \text{and} \quad h = N' + 1, \cdots, N' + N'')
\end{aligned} \tag{32}$$

and these are the desired approximate equations. They are in general non-linear as are the original set (1). However the approximate equations are usually more convenient for some elementary treatment or for numerical solution or else qualitative investigation. There is also this advantage that the equations of the first two sets (32) are independent of the u's and so may be integrated to give the expressions of the ξ's and a's; the u's are then determined by quadratures from the third set (32).

If the φ's are independent of t we can in particular discover the steady quasi-periodic oscillations postulating the ξ's, a's and the derivatives $du_h/dt = \omega_h + d\theta_h/dt$ to be constant. Putting $d\theta_h/dt = \omega_h C_h$ the equa-

tions (32) become

$$\kappa_g\xi_g + \frac{1}{(2\pi)^{N''}\Delta'(\kappa_g)} \times \sum_{k=1}^{n} B_k^{(g)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k du_{N'+1} \cdots du_{N'+N''} = 0,$$

$$\epsilon_h a_h + \frac{1}{2^{N''-1}\pi^{N''}} \operatorname{Re} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \times \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_{N'+1} \cdots du_{N'+N''} = 0,$$

$$-C_h\omega_h a_h + \frac{1}{2^{N''-1}\pi^{N''}} \operatorname{Im} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \times \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_{N'+1} \cdots du_{N'+N''} = 0,$$

$$(g = 1, \cdots, N' \quad \text{and} \quad h = N'+1, \cdots, N'+N''). \quad (33)$$

These are $N' + 2N''$ finite but in general also non-linear equations in $N' + N''$ amplitudes ξ_g, a_h and N'' quantities C_h which are frequency corrections since the expressions of x_j^{ν} can be written in the case considered as

$$x_j^{\nu} = \sum_g X_{j\nu}^{(g)}\xi_g + \sum_h N_{j\nu}^{(h)} a_h \cos\left[\omega_h(1+C_h)t + \delta_h + \gamma_j^{(h)} + v\zeta_h\right], \quad (34)$$

the δ's being arbitrary constants.

The equations (33) may be regarded as a direct generalization of the equations for a simple periodic process obtained by H. Poincaré's method in the previous communication of the author [1]. To account for this case here we note that when all a_ν except a certain a_h, vanish we have

$$x_j^{\nu} = \sum_g X_{j\nu}^{(g)}\xi_g + N_{j\nu}^{(h)} a_h \cos\left[\omega_h(1+C_h)t + \delta_h + \gamma_j^{(h)} + v\zeta_h\right] \quad (35)$$

and

$$\int_{-\pi}^{+\pi} \cdots \int_{-\pi}^{+\pi} \varphi_k e^{-iu_\nu} du_{N'+1} \cdots du_{N'+N''} = 0 \quad \text{for} \quad \nu \neq h.$$

The formulae (35) represent a simple periodic solution, the $N' + 2$ constants $\xi_1, \cdots, \xi_{N'}, a_h, C_h$ being determined by the $N' + 2$ equations

$$\kappa_g\xi_g + \frac{1}{2\pi\Delta'(\kappa_g)} \sum_{k=1}^{n} B_k^{(g)} \int_{-\pi}^{+\pi} \varphi_k du_h = 0 \qquad (g = 1, \cdots, N'),$$

$$\epsilon_h a_h + \frac{1}{\pi} \operatorname{Re} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_h = 0,$$

$$-C_h a_h \omega_h + \frac{1}{\pi} \operatorname{Im} \frac{1}{\Delta'(\epsilon_h + i\omega_h)} \sum_{k=1}^{n} B_k^{(h)} \int_{-\pi}^{+\pi} \varphi_k e^{-iu_h} du_h = 0. \quad (36)$$

In the still more special case when the characteristic equation has no real roots and also $\epsilon_k = 0$, there are only two constants $a_k = A_k$, C_k to determine; replacing $B_k^{(k)}$ with their expressions (7) it is seen that the two last equations (36) can be obtained from

$$A_k s_k \left[\frac{\partial \Delta(D)}{\partial D} DC_k \right]_{D=i\omega_k} = \frac{1}{\pi} \sum_{k=1}^{n} F_{k,\, m(k)}(i\omega_k) \int_{-\pi}^{+\pi} \varphi_k e^{-iu_k} du_k$$

separating into real and imaginary parts. But this equation coincides with the relation (28) of the paper cited if we put there $\partial\Delta/\partial\mu = 0$ considering that the linear terms in (1) and (26) above are the same; besides that, in order to bring the notations in agreement, we must replace C_k, φ_k respectively by μC, $\mu\varphi_k$ and then cancel the common factor μ.

3. *Equations of Motion of an Automatically Controlled System Possessing Inertia.* We shall now apply the preceding theory to an automatically controlled system possessing inertia as considered in a series of articles in the "Engineer" [7] as well as by W. Oppelt [8], V. Kotelnikov [9], N. Minorsky [10] and others. The equations of motion can be written as

$$\begin{aligned} T^2\ddot{\varphi} &= -U\dot{\varphi} - k\varphi - \mu \qquad \text{(system to be controlled)}, \\ V\dot{\mu} &= H\left(l\varphi + X\dot{\varphi} + Y^2\ddot{\varphi} - \frac{1}{m}\mu \right) \qquad \text{(servo-motor)}, \end{aligned} \tag{37}$$

where φ denotes the departure from the pre-selected state and μ is the coordinate of the regulating member operated by the servo-motor. Capital letters T, U, V, X, Y denote constants having the dimension of time whereas k, l, m are dimensionless. In essence it is M. Tolle's [11] system of notation. The term $T^2\ddot{\varphi}$ represents the inertia, $-U\dot{\varphi}$ the natural damping, $-k\varphi$ the restoring force and $-\mu$ the control action. The function H, in general non-linear, is the characteristic of the servo-motor. We suppose it to be a monotonic increasing odd function vanishing with its argument

$$\psi = l\varphi + X\dot{\varphi} + Y^2\ddot{\varphi} - \frac{1}{m}\mu, \tag{38}$$

which measures the opening of the piston valve of the servo-motor supposed self-braking so that its inertia as well as that of the regulating organ may be neglected and its velocity assumed to be determined directly by ψ. The graph of $H(\psi)$ is of the type shown in Fig. 1. The constant V is chosen so as to make the maximum value of $H(\psi)$ equal to unity; it may be referred to as the time of the servo-motor. The terms $l\varphi$, $X\dot{\varphi}$, $Y^2\ddot{\varphi}$ of ψ account for the signals transmitted to the valve from the sensitive elements which respond to the departure of the system, its velocity and acceleration; the last term μ/m expresses the dis-

placement of the regulating member referred back to the valve by the follow-up linkage and m is the corresponding transmission ratio.

The constants T^2, U, V, m are supposed positive; k is positive for a natural stable system but it is equal to zero when the system is neutral and negative when unstable. The negative coefficient l and the negative constant X are practically of little use but we need no preconception relative to their signs or to the sign of Y^2.

Problems of the kind considered are sometimes treated by means of "retarded" functions of the argument $t - \Delta t$, where Δt is the time lag due to the servo-motor. To account for this lag by means of an appropriate differential equation seems more natural and it has been done in linear treatment as for instance by W. Oppelt [8] and H. Weiss [12].

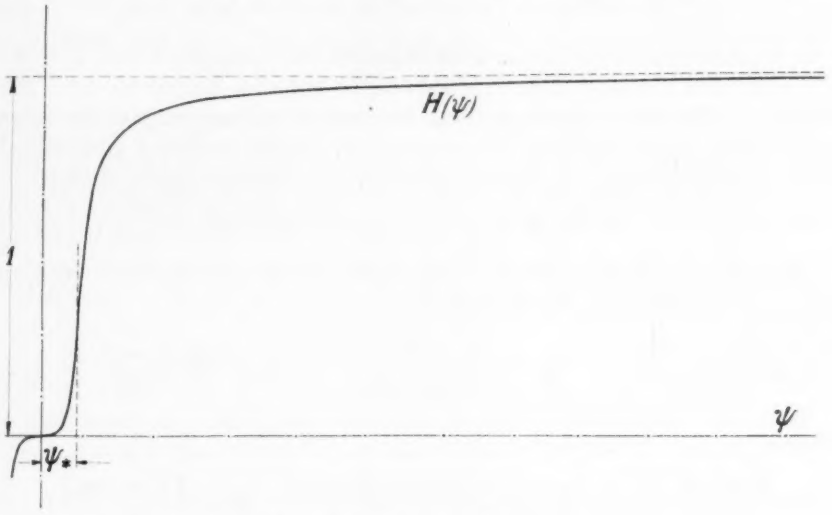

FIG. 1. Characteristic curve of the servo-motor.

The non-linearity of the characteristic is however by no means inessential for in it lies the only explanation of such a phenomenon as the possible maintained oscillations.

We can use the first equation (37) to eliminate $\ddot{\varphi}$ from the expression of ψ and represent it as

$$\psi = \frac{lT^2 - kY^2}{T^2}\varphi + \frac{XT^2 - UY^2}{T^2}\dot{\varphi} - \frac{T^2 + mY^2}{mT^2}\mu. \qquad (39)$$

Introducing in place of φ, μ, t new variables by the formulae

$$x = \frac{1}{p}\varphi, \qquad y = \frac{1}{q}\psi = \frac{1}{qmT^2}[m(XT^2 - UY^2)\dot{\varphi} + m(lT^2 - kY^2)\varphi - (T^2 + mY^2)\mu], \qquad \tau = \frac{1}{r}t \quad (40)$$

in which p, q, r are constants to be specified further, we obtain

$$\frac{T^2+mY^2}{m}\frac{p}{qr^2}\frac{d^2x}{d\tau^2}+\frac{U+mX}{m}\frac{p}{qr}\frac{dx}{d\tau}+\frac{k+lm}{m}\frac{p}{q}x-y=0,$$
$$-V\left[\frac{XT^2-UY^2}{T^2}\frac{p}{qr}\frac{d^2x}{d\tau^2}+\frac{lT^2-kY^2}{T^2}\frac{p}{q}\frac{dx}{d\tau}-\frac{dy}{d\tau}\right]$$
$$+\frac{T^2+mY^2}{T^2m}\frac{r}{q}H(qy)=0. \quad (41)$$

We put first $p=q=r=1$. The second equation shows that for an ideal servo-motor with no time-lag ($V=0$) it must be continually $y=0$ and then the first equation becomes

$$(T^2+mY^2)\ddot{x}+(U+mX)\dot{x}+(k+lm)x=0. \quad (42)$$

It determines linear damped oscillations and it appears that T^2+mY^2, $U+mX$, $k+lm$ measure the complete inertia, damping and static stability. The first terms of these binomials correspond to the respective natural properties and the second represent artificial addenda due to the control device; in all the following it will be supposed that

$$T^2+mY^2>0, \qquad k+lm>0. \quad (43)$$

Another conclusion relative especially to the acceleration control is arrived at introducing the notations

$$l_e=\frac{lT^2-kY^2}{T^2}, \qquad m_e=\frac{mT^2}{T^2+mY^2}, \qquad X_e=\frac{XT^2-UY^2}{T^2}, \quad (44)$$

so that

$$l=l_e+\frac{k(m-m_e)}{mm_e}, \qquad X=X_e+\frac{U(m-m_e)}{mm_e}, \qquad Y^2=\frac{T^2(m-m_e)}{mm_e}. \quad (45)$$

The formula (39) becomes

$$\psi=l_e\varphi+X_e\dot{\varphi}-\frac{1}{m_e}\mu \quad (46)$$

and the same is obtained from (38) for a system with $l=l_e$, $m=m_e$, $X=X_e$, $Y=0$. It follows that so long as continuous external perturbations are not considered the introduction of the acceleration control is equivalent to an alteration of the parameters l, m, X in a device without such control. We can suppose even $m\to\infty$ and still obtain any desired equivalent parameters l_e, m_e, X_e by giving to l, X, Y^2 the finite limiting values which result from (45). In this case the acceleration control completely replaces the follow-up linkage materially not brought about in the system.

To reduce the number of the parameters we assume from now on a

second determination of p, q, r, viz.

$$p = 10\psi_* \frac{m}{k + lm}, \qquad q = 10\psi_*, \qquad r = \sqrt{\frac{T^2 + mY^2}{k + lm}}, \tag{47}$$

the symbol ψ_* denoting the middle abscissa of the interval of the rapid increase of $H(\psi)$. Putting

$$u = \frac{U}{T^2}\sqrt{\frac{T^2 + mY^2}{k + lm}}, \qquad \alpha = \frac{k}{k + lm}\frac{T^2 + mY^2}{T^2},$$

$$\beta = \frac{U + mX}{\sqrt{(k + lm)(T^2 + mY^2)}}, \qquad \gamma = \frac{1}{10\psi_*}\frac{(T^2 + mY^2)^{\frac{1}{2}}}{T^2 V m \sqrt{k + lm}},$$

$$H(10\psi_* y) = F(y) \tag{48}$$

and denoting by D the operator $d/d\tau$, the equations (41) become

$$\begin{aligned} (D^2 + \beta D + 1)x - y &= 0, \\ -[(\beta - u)D^2 + (1 - \alpha)D]x + Dy + \gamma F(y) &= 0. \end{aligned} \tag{49}$$

The graph of $F(y)$ is of the same type as that of $H(\psi)$ but the quantity y_* corresponding to ψ_* has a standard value $\psi_*/q = 0.1$.

4. *Approximate Linearisation.* We shall try to represent $F(y)$ approximately as a binomial $g + hy$ in the range of the arguments considered, that is to replace a portion of the characteristic curve by a suitable secant. This representation is to be adapted for the study of the steady periodic vibrations since their existence and properties are determinative for the motion in general; the actual calculation of the amplitudes and the coefficients of the binomial will be carried out later on. For the time being we treat g, h as given and write (49) in the form

$$\begin{aligned} (D^2 + \beta D + 1)(x - x^*) - (y - y^*) &= \varphi_1, \\ -[(\beta - u)D^2 + (1 - \alpha)D](x - x^*) + (D + \gamma h)(y - y^*) &= \varphi_2 \end{aligned} \tag{50}$$

with

$$x^* = y^* = -\frac{g}{h}, \tag{51}$$

$$\varphi_1 = 0, \qquad \varphi_2 = \gamma[g + hy - F(y)] = \gamma[h(y - y^*) - F(y)]; \tag{52}$$

the function φ_2 is a small non-linear correction.

Cancelling φ_1, φ_2 we obtain linear homogenous equations with constant coefficients in the unknowns $x - x^*$, $y - y^*$. These equations will be termed simplified; their operational determinant is

$$\Delta = \begin{vmatrix} D^2 + \beta D + 1, & -1 \\ -(\beta - u)D^2 - (1 - \alpha)D, & D + \gamma h \end{vmatrix} = D^3 + d_1 D^2 + d_2 D + d_3, \tag{53}$$

wherein

$$d_1 = \gamma h + u, \qquad d_2 = \beta\gamma h + \alpha, \qquad d_3 = \gamma h. \tag{54}$$

The Routh's conditions for the characteristic equation $\Delta = 0$ can be written as

$$d_1 = \gamma h + u > 0, \qquad d_1 d_2 - d_3 = e_0 h^2 + e_1 h + e_2 > 0, \qquad d_3 = \gamma h > 0 \tag{55}$$

with

$$e_0 = \beta\gamma^2, \qquad e_1 = \gamma(\alpha + u\beta - 1), \qquad e_2 = u\alpha. \tag{56}$$

The two roots of the polynomia $e_0 h^2 + e_1 h + e_2$ are

$$h_1, h_2 = \frac{1}{2\beta\gamma}(-\alpha - u\beta + 1 \mp R), \tag{57}$$

where

$$R^2 = (\alpha - 1)^2 - 2(\alpha + 1)u\beta + (u\beta)^2 \tag{58}$$

and R is the positive value of $\sqrt{R^2}$.

Periodic solutions of the simplified equations are possible when the characteristic equation admits of a pair of pure imaginary conjugate roots. It is known that for this it is necessary that the last Routh's inequality but one becomes an equality, that is

$$d_1 d_2 - d_3 = e_0 h^2 + e_1 h + e_2 = 0 \tag{59}$$

and h must have one of the values $h_i (i = 1, 2)$. If this is the case, then, using (59), we obtain

$$\Delta = D^3 + d_1 D^2 + d_2 D + d_1 d_2 = D^2(D + d_1) + d_2(D + d_1) = (D + \kappa)(D^2 + \omega^2),$$

where

$$\kappa = d_1 = \gamma h_i + u = \frac{1}{2\beta}(-\alpha + u\beta + 1 \mp R), \tag{60}$$

$$\omega^2 = d_2 = \frac{d_3}{d_1} = \beta\gamma h_i + \alpha = \frac{1}{2}(\alpha - u\beta + 1 \mp R). \tag{61}$$

The roots of Δ are thus $-\kappa, \pm i\omega$. If $d_3/d_1 = \gamma h_i/(\gamma h_i + u) > 0$ and, in particular, if h_i is positive, then ω is real and represents the frequency of the periodic vibration with respect to the dimensionless "time" τ. It remains to find out under what conditions the values h_i may serve to construct an acceptable linear approximation for $F(y)$ so that the periodic solutions of the simplified equations would represent approximately such solutions for the original non-linear set.

5. *The Steady States of Vibration and the Equations of Transitory Motions in Their Vicinity.* We shall now apply the transformation to normal coordinates and the approximate equations of the general theory to the pseudo-linear system (50) supposing h to have one of the values h_i and putting

$$x_1 = x - x^*, \qquad x_2 = y - y^*, \tag{62}$$

$$N' = N'' = 1, \qquad \kappa_1 = -\kappa, \qquad \epsilon_2 = 0, \qquad \omega_2 = c_2 = \omega, \qquad \zeta_2 = \frac{\pi}{2}. \tag{63}$$

If we assume

$$s_1 = s_2 = 1, \qquad l(1) = l(2) = 1, \qquad m(1) = m(2) = 2, \tag{64}$$

then we obtain from (6), (7)

$$X_1^{(1)} = \frac{1}{B_2^{(1)}} = \frac{1}{Q}, \qquad X_2^{(1)} = 1, \tag{65}$$

where by (60), (61),

$$Q = \kappa^2 - \beta\kappa + 1 = \kappa^2 + \omega^2 \pm R; \tag{66}$$

similarly

$$C_1^{(2)} = N_1^{(2)} \exp i\gamma_1^{(2)} = \frac{1}{B_2^{(2)}} = \frac{1}{-\omega^2 + 1 + i\beta\omega},$$
$$C_2^{(2)} = N_2^{(2)} \exp i\gamma_2^{(2)} = 1. \tag{67}$$

Setting

$$\xi_1 = a, \qquad a_2 = C, \qquad u_2 = u = \omega\tau + \theta, \tag{68}$$

we have by (19), (29) the transformation

$$\begin{aligned} x &= x^* + X_1^{(1)}a + N_1^{(2)}C\cos(u + \gamma_1^{(2)}), \\ \dot{x} &= -X_1^{(1)}\kappa a - N_1^{(2)}(\omega C \sin(u + \gamma_1^{(2)}), \\ y &= y^* + a + C\cos u \end{aligned} \tag{69}$$

to new coordinates a, C, u.

Next we note that

$$\begin{aligned} \Delta(D) &= (D + \kappa)(D - i\omega)(D + i\omega), \\ \Delta'(-\kappa) &= \kappa^2 + \omega^2, \qquad \Delta'(i\omega) = 2\omega(-\omega + i\kappa), \end{aligned} \tag{70}$$

$$\frac{B_2^{(2)}}{\Delta'(i\omega)} = \frac{(-\omega^2 + 1 + i\beta\omega)(-\omega - i\kappa)}{2\omega(\kappa^2 + \omega^2)} \tag{71}$$

and remember that $\varphi_1 = 0$; then the equations (30), (31) for the problem in hand may be written as

$$\begin{aligned} \frac{da}{d\tau} &= -\kappa a + \frac{Q}{\kappa^2+\omega^2}\varphi_2(y^*+a+C\cos u), \\ \frac{dC}{d\tau} &= \mathrm{Re}\left\{\frac{\omega(\omega^2-1+\beta\kappa)+i[\kappa(\omega^2-1)-\beta\omega^2]}{\omega(\kappa^2+\omega^2)}\right\}e^{-iu}\varphi_2(y^*+a+C\cos u), \\ \frac{du}{d\tau} &= \omega + \frac{1}{C}\mathrm{Im}\{\mathrm{idem}\} \end{aligned} \tag{72}$$

and we have by (60), (61)

$$\omega^2 - 1 + \beta\kappa = \mp R. \tag{73}$$

To give (72) a final form we obtain with the aid of (52)

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi}\varphi_2(y^*+a+C\cos u)du = \gamma\left[\, h_1 a - \frac{1}{2\pi}\int_{-\pi}^{+\pi} F(y^*+a+C\cos u)du \,\right],$$

$$\frac{1}{\pi}\int_{-\pi}^{+\pi}\varphi_2(y^*+a+C\cos u)\cos u du = \gamma\left[\, h_1 C - \frac{1}{\pi}\int_{-\pi}^{+\pi} F(y^*+a+C\cos u)\cos u du \,\right]$$

and also, putting $\varphi_2(y) = \chi'(y)$,

$$\frac{1}{\pi}\int_{-\pi}^{+\pi}\varphi_2(y^*+a+C\cos u)\sin u du = -\frac{1}{\pi C}\int_{-\pi}^{+\pi}\chi'(y^*+a+C\cos u)d(C\cos u)$$

$$= -\frac{1}{\pi C}[\chi(y^*+a+C\cos u)]_{-\pi}^{+\pi} = 0.$$

With this and replacing the right-hand members of (72) by their mean values we have

$$\frac{da}{d\tau} = A(a, C), \qquad \frac{dC}{d\tau} = B(a, C), \tag{74}$$

$$\frac{du}{d\tau} = \omega + \frac{\kappa(\omega^2-1)-\beta\omega^2}{2C\omega(\kappa^2+\omega^2)}\gamma\left[h_1 C - \frac{1}{\pi}\int_{-\pi}^{+\pi} F(y^*+a+C\cos u)\cos u du\right], \tag{75}$$

where

$$A(a, C) = -\kappa a + \frac{Q\gamma}{\kappa^2+\omega^2}\left[h_1 a - \frac{1}{2\pi}\int_{-\pi}^{+\pi} F(y^*+a+C\cos u)du\right],$$

$$B(a, C) = \frac{\mp R\gamma}{2(\kappa^2+\omega^2)}\left[h_1 C - \frac{1}{\pi}\int_{-\pi}^{+\pi} F(y^*+a+C\cos u)\cos u du\right]. \tag{76}$$

As a first application we use these equations to determine the steady states corresponding to constant values a^*, C^*. For these it must be $A(a^*, C^*) = B(a^*, C^*) = 0$. On the other hand to ensure better local approximation for such states and in their vicinity we impose the condition that all mean values in (76) arising from those terms of exact equations which contained the non-linear correction φ_2 should vanish when $a = a^*$, $C = C^*$:

$$h_1 a^* - \frac{1}{2\pi}\int_{-\pi}^{+\pi} F(y^*+a^*+C^*\cos u)du = 0,$$

$$h_1 C^* - \frac{1}{\pi}\int_{-\pi}^{+\pi} F(y^*+a^*+C^*\cos u)\cos u du = 0.$$

It follows that $\kappa a^* = 0$; from (58), (60) it is seen that κ cannot vanish for a finite β and $u > 0$. So it must be $a^* = 0$ and

$$\int_{-\pi}^{+\pi} F(y^* + C^* \cos u)du = 0,$$

$$h_1 C^* - \frac{1}{\pi}\int_{-\pi}^{+\pi} F(y^* + C^* \cos u) \cos u du = 0.$$

The function $F(y)$ being odd and increasing the first relation is only possible if $y^* = 0$ though it would be otherwise in the case of an unsymmetrical characteristic. The second relation then becomes

$$J(C^*) = h_1 C^* \tag{77}$$

with

$$J(C) = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(C \cos u) \cos u du \tag{78}$$

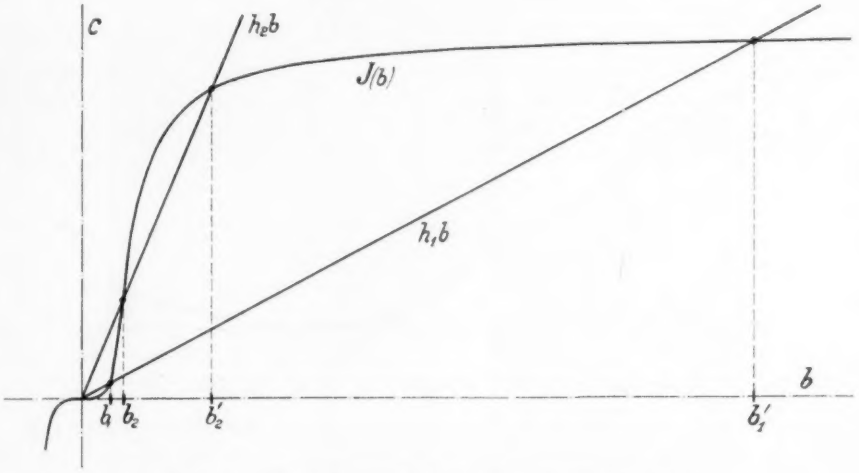

FIG. 2. Determination of the stationary amplitudes.

and in (77) we have a finite equation, in general non-linear, for the determination of the stationary amplitudes C^*. To $a = a^* = 0$ and $C = C^*$ corresponds, as is seen from (75), the constant value ω of $du/d\tau$; consequently we have in virtue of (69) a periodic solution in the original unknowns x, y. It determines steady undamped vibrations and, when stable, they are termed maintained.

According to the general remark at the end of Sect. 2 we might also obtain the periodic solutions by H. Poincaré's method using the same simplified equations as above with $g = x^* = y^* = 0$, $h = h_1$. The "generating" amplitudes would coincide with the solutions C^* of (77) and the first frequency correction would be equal to zero.

6. *Dependence of the Steady Vibrations on the System's Parameters.* The equation (77) may be solved graphically as shown in Fig. 2, the

stationary amplitudes C^* being determined as the abscissae of the points of intersection of the curve $c = J(C)$ with the secant $c = h_i C$. Such points can be different from the origin only if $0 < h_i < h_*$, where h_* corresponds to the tangent to the curve $c = J(C)$ drawn through the origin. The existence and behaviour of the periodic solutions thus depend on the nature of the roots h_i of the polynomial $e_0 h^2 + e_1 h + e_2$. To study them in their dependence on the parameters of the system we interpret the quantities

$$\xi = \alpha = \frac{k(T^2 + mY^2)}{T^2(k + lm)}, \qquad \eta = u\beta = \frac{U(U + mX)}{T^2(k + lm)} \tag{79}$$

as the coordinates of a point in the diagram (Fig. 3). The parameters γ, u are regarded as absolute constants.

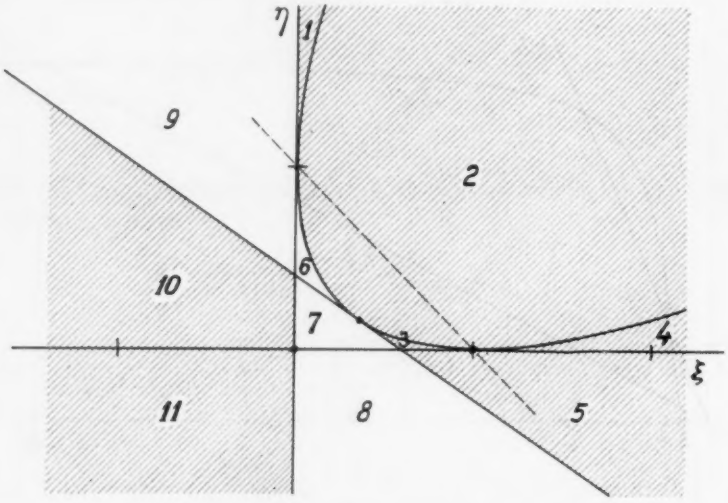

FIG. 3. Periodic solutions and the stability of the system.

In virtue of (58) the equation $R^2 = 0$ represents a conic; writing it in the form

$$(-\xi + \eta)^2 - 2(\xi + \eta - \tfrac{1}{2}) = 0, \tag{80}$$

we recognize it as the equation of a parabola, whose axis is the bisector $-\xi + \eta = 0$ of the angle between the coordinate axes. These axes touch the curve at the points (1, 0) and (0, 1) and on them the coefficients $e_0 = (\gamma^2/u)\eta$ and $e_2 = u\xi$ respectively change signs. The coefficient e_1 changes sign on the straight line

$$\frac{e_1}{\gamma} = \xi + \eta - 1 = 0 \tag{81}$$

passing through the contact points (1, 0) and (0, 1).

We also ask for the locus of the points where one of h_i assumes the value h_* so that $e_0h_*^2 + e_1h_* + e_2 = 0$; or, by (56),

$$(\gamma h_* + u)(u\xi + \gamma h_*\eta) = u\gamma h_*. \tag{82}$$

It is the equation of a straight line tangent to the parabola at the point with the coordinates

$$\xi = \left(\frac{\gamma h_*}{\gamma h_* + u}\right)^2, \qquad \eta = \left(\frac{u}{\gamma h_* + u}\right)^2. \tag{83}$$

When $\gamma h_* = 0, \infty$ this tangent coincides respectively with the η- and ξ-axis and the contact point goes to (0, 1) or to (1, 0). Accordingly, one of the roots h_i becomes infinite on the ξ-axis whereas on the η-axis one root vanishes.

The parabola, the two axes and the tangent (82) divide the plane into eleven domains shown in Fig. 3, the signs of e_0, e_1, e_2, R^2 and the roots h_i in these being as indicated in the table.

1	$e_0 > 0, e_1 > 0, e_2 > 0, R^2 > 0$	$h_1 < 0, h_2 < 0$	absolute convergence
2	$e_0 > 0, \quad e_2 > 0, R^2 < 0$	h_1, h_2 complex	
3	$e_0 > 0, e_1 < 0, e_2 > 0, R^2 > 0$	$h_* < h_1, h_* < h_2$	
4	$e_0 > 0, e_1 > 0, e_2 > 0, R^2 > 0$	$h_1 < 0, h_2 < 0$	
5	$e_0 < 0, \quad e_2 > 0, R^2 > 0$	$h_* < h_1, h_2 < 0$	
6	$e_0 > 0, e_1 < 0, e_2 > 0, R^2 > 0$	$0 < h_1 < h_*, 0 < h_2 < h_*$	steady vibrations
7	$e_0 > 0, e_1 < 0, e_2 > 0, R^2 > 0$	$0 < h_1 < h_*, h_* < h_2$	
8	$e_0 < 0, \quad e_2 > 0, R^2 > 0$	$0 < h_1 < h_*, h_2 < 0$	
9	$e_0 > 0, \quad e_2 < 0, R^2 > 0$	$h_1 < 0, 0 < h_2 < h_*$	
10	$e_0 > 0, \quad e_2 < 0, R^2 > 0$	$h_1 < 0, h^* < h_2$	absolute divergence
11	$e_0 < 0, e_1 < 0, e_2 < 0, R^2 > 0$	$h_1 < 0, h_2 < 0$	

It appears that the steady undamped vibrations are only possible in the domains 6, 7, 8, 9; their properties essentially depend on their stability studied in Sect. 7. In the two regions formed by the groups of domains 1, 2, 3, 4, 5 and, respectively, 10, 11 (hatched) no periodic vibrations are possible. In the first instance we have $\alpha > 0$ so that also $k > 0$ and the system is statically stable; no steady state different from the equilibrium being possible the vibrations are convergent for all initial perturbations. In the second instance $k < 0$ and the vibrations are absolutely divergent. The existence of the intermediate (unhatched) region of steady vibrations is characteristic for non-linear systems. In fact let $V \to \infty$ which is to say that the effectiveness of the control becomes infinitesimal; the second equation (37) ultimately gives $\dot{\mu} = 0$ and we can assume $\mu = 0$ so that the motion is determined by the linear equation $T^2\ddot{\varphi} + U\dot{\varphi} + k\varphi = 0$. On the other hand, in virtue of (48) we have $\gamma \to 0$ and the tangent (82) tends to coincidence with the η-axis; the unhatched space contracts to the same axis and in the limit the two hatched regions overspread the whole plane.

In applications we must refer to the natural parameters. For instance we may fix T, U, V, X, Y, k, l and cause the transmission ratio m to assume a range of positive values. Eliminating m from (79) it is seen that the point (ξ, η) then describes a segment of the straight line

$$T^2U(lU - kX)\xi - kT^2(lT^2 - kY^2)\eta + kU(XT^2 - UY^2) = 0 \quad (84)$$

and we must trace its position with respect to the domains of the $\xi\eta$-plane taking into consideration that the tangent (82) also changes its position with m.

7. *Stability of the Steady Vibrations.* To settle the question as to the stability of the steady states we must investigate the disturbed motion $a = a^* + \delta a$, $C = C^* + \delta C$ and we have a ready instrument for this in the approximate equations (74). The consideration of small disturbances δa, δC being sufficient we use the equations in variations

$$\frac{d(\delta a)}{d\tau} = \left(\frac{\partial A}{\partial a}\right)^* \delta a + \left(\frac{\partial A}{\partial C}\right)^* \delta C, \qquad \frac{d(\delta C)}{d\tau} = \left(\frac{\partial B}{\partial a}\right)^* \delta a + \left(\frac{\partial B}{\partial C}\right)^* \delta C \quad (85)$$

obtained in the usual way by expanding in Taylor series and neglecting the higher terms. $(\ )^*$ symbolizes the substitution of the values $a = a^* = 0$, $C = C^*$ and namely

$$\begin{aligned}
\left(\frac{\partial A}{\partial a}\right)^* &= -\kappa + \frac{Q\gamma}{\kappa^2 + \omega^2}\left[h_i - \frac{1}{2\pi}\int_{-\pi}^{+\pi} F'(C^* \cos u)du\right], \\
\left(\frac{\partial A}{\partial C}\right)^* &= -\frac{Q\gamma}{\kappa^2 + \omega^2}\frac{1}{2\pi}\int_{-\pi}^{+\pi} F'(C^* \cos u)\cos u du, \\
\left(\frac{\partial B}{\partial a}\right)^* &= \frac{\pm R\gamma}{\kappa^2 + \omega^2}\frac{1}{2\pi}\int_{-\pi}^{+\pi} F'(C^* \cos u)\cos u du, \\
\left(\frac{\partial B}{\partial C}\right)^* &= \frac{\mp R\gamma}{2(\kappa^2 + \omega^2)}\left[h_i - \frac{1}{\pi}\int_{-\pi}^{+\pi} F'(C^* \cos u)\cos^2 u du\right].
\end{aligned} \quad (86)$$

Differentiating (78) with respect to C, using an integration by parts and taking (77) into consideration, we obtain

$$J'(C) = \frac{1}{\pi}\int_{-\pi}^{+\pi} F'(C\cos u)\cos^2 u du,$$

$$\begin{aligned}
\frac{1}{\pi}\int_{-\pi}^{+\pi} F'(C^*\cos u)du &= \frac{1}{\pi}\int_{-\pi}^{+\pi} F'(C^*\cos u)(\cos^2 u + \sin^2 u)du \\
&= J'(C^*) - \frac{1}{\pi C^*}\left[F(C^*\cos u)\sin u\right]_{-\pi}^{+\pi} \\
&\quad + \frac{1}{\pi C^*}\int_{-\pi}^{+\pi} F(C^*\cos u)\cos u du = J'(C^*) + h_i
\end{aligned}$$

and therefore

$$\left(\frac{\partial A}{\partial a}\right)^* = -\kappa + \frac{Q\gamma}{2(\kappa^2+\omega^2)}[h_i - J'(C^*)],$$
$$\left(\frac{\partial B}{\partial C}\right)^* = \frac{\mp R\gamma}{2(\kappa^2+\omega^2)}[h_i - J'(C^*)]. \tag{87}$$

On the other hand, $F(y)$ being odd and $F'(y)$ even the integral in the second and third expressions (86) vanishes and so

$$\left(\frac{\partial A}{\partial C}\right)^* = \left(\frac{\partial B}{\partial a}\right)^* = 0. \tag{88}$$

The roots of the characteristic equation of the system in variations (85) are thus real numbers $(\partial A/\partial a)^*$, $(\partial B/\partial C)^*$ and if both are negative then the steady state is stable.

Using (60), (66) it is seen that the sum

$$\left(\frac{\partial A}{\partial a}\right)^* + \left(\frac{\partial B}{\partial C}\right)^* = -\kappa + \frac{(Q \mp R)\gamma}{2(\kappa^2+\omega^2)}[h_i - J'(C^*)]$$
$$= -(\gamma h_i + u) + \tfrac{1}{2}\gamma[h_i - J'(C^*)] = -u - \tfrac{1}{2}\gamma[h_i + J'(C^*)] \tag{89}$$

is always negative since for the steady states $h_i > 0$, $J'(C^*) > 0$. Consequently the condition of the stability can be given the form

$$\left(\frac{\partial A}{\partial a}\right)^*\left(\frac{\partial B}{\partial C}\right)^* = \frac{\pm R\gamma}{4(\kappa^2+\omega^2)^2}[h_i - J'(C^*)][S + Q\gamma J'(C^*)] > 0, \tag{90}$$

whereby (60), (66)

$$S = 2\kappa(\kappa^2+\omega^2) - Q\gamma h_i = (2\gamma h_i + 2u)(\kappa^2+\omega^2)$$
$$- \gamma h_i(\kappa^2+\omega^2 \pm R) = (\kappa^2+\omega^2)(\gamma h_i + 2u) \mp \gamma h_i R.$$

If condition (90) is fulfilled then, according to the classification of Poincaré [13] the singular point (a^*, C^*) of the equations (74) is a stable node; in the opposite case it is a saddle-point but never a focal point.

Using (57), (58), (60), (61) we can express S in terms of u, α, β and the result of this calculation, somewhat lengthy, is

$$2\beta^3 S = u(-\alpha + 2u^2 - 2)\beta^3 + [\alpha^2 - (5u^2+3)\alpha + 2]\beta^2$$
$$+ u(\alpha - 1)(4\alpha - 1)\beta + (-\alpha + 1)^3 \mp [(-\alpha + 2u^2 + 2)\beta^2$$
$$+ u(-3\alpha + 2)\beta + (-\alpha + 1)^2]R. \tag{91}$$

The question of the stability can always be answered for given numerical values with the aid of the formulae obtained; however, for the general discussion we now confine ourselves to the simple case of sufficiently small positive or negative α. Determining the signs of the quantities considered when $\alpha = 0$ we then use the circumstance that,

in virtue of the continuity, the signs will be the same in the neighbourhood of that value. We begin with the following case.

1. $|\alpha|$ small, $u\beta > 1$ (the boundary of the domains 1 and 9 and its neighbourhood, Fig. 3).

Using throughout the subscripts 1, 2 for the values corresponding respectively to h_1 (upper sign before R) and h_2 (lower sign) we have when $\alpha = 0$:

$$R = u\beta - 1, \qquad h_1 = -\frac{u\beta - 1}{\beta\gamma} < 0, \qquad h_2 = 0.$$

On the η-axis there are thus no periodic solutions and so is the case with the domain 1 where h_2 becomes negative.

But in 9 this root satisfies the inequality $0 < h_2 < h_*$ and it is seen from Fig. 2 that here we have two periodic solutions with amplitudes $C^* = C_2,\ C_2'$ for which respectively $J'(C_2) > h_2,\ J'(C_2') < h_2$ (the figure properly corresponds to the following case II and for the present only the upper secant $c = h_2 C$ must be taken into consideration). Putting again $\alpha = 0$ we calculate

$$\kappa_2 = \gamma h_2 + u = u, \qquad Q_2 = u^2 - \beta u + 1, \qquad S_2 = 2u^3 > 0.$$

If

$$1 < u\beta < 1 + u^2$$

then

$$Q_2 > 0, \qquad S_2 + Q_2\gamma J'(C_2) > 0, \qquad S_2 + Q_2\gamma J'(C_2') > 0$$

and these inequalities hold in the adjoining part of 9. The condition (90) in which we must take the lower sign is fulfilled for C_2 but not for C_2'. The first amplitude is stable, the second unstable and we sum up that here we have *maintained vibrations of small amplitude.*

If $1 + u^2 < u\beta$ then Q_2 becomes negative when $\alpha = 0$. However, so long as α is negative but sufficiently small in absolute magnitude the root h_2 and both $J'(C_2)$ and $J'(C_2')$ are also small. The result is that the first positive terms in $S_2 + Q_2\gamma J'(C_2)$, $S_2 + Q_2\gamma J'(C_2')$ are dominant and the foregoing conclusion concerning the stability remains true. Farther from the η-axis, however, the amplitudes C_2, C_2' may well be both unstable.

Supposing now that $u\beta < 1$ we put $\alpha = 0$, calculate

$$R = 1 - u\beta, \qquad h_1 = 0, \qquad h_2 = \frac{1 - u\beta}{\beta\gamma},$$

$$\kappa_1 = \gamma h_1 + u = u, \qquad Q_1 = u^2 - u\beta + 1, \qquad S_1 = 2u^3,$$

$$\kappa_2 = \gamma h_2 + u = \frac{1}{\beta}, \qquad Q_2 = \frac{1}{\beta^2}, \qquad \beta^2 S_2 = 2\beta^2(1 - u\beta) + 1 + u\beta$$

and consider separately two following cases.

II. $|\alpha|$ small, $u/(u + \gamma h_*) < u\beta < 1$ (the boundary of the domains 9 and 6 and its neighborhood).

When $\alpha = 0$ we have

$$h_1 = 0, \qquad Q_1 > 0, \qquad S_1 > 0, \qquad 0 < h_2 < h^*, \qquad Q_2 > 0, \qquad S_2 > 0$$

and so on the η-axis we have stationary amplitudes C_2, C_2' appropriate to h_2. Of these, as before, C_2 is stable, C_2' unstable and the system admits of *maintained vibrations of small amplitude*. The same circumstances presents itself in the adjoining part of 9 where h_1 becomes negative.

But on the other side of the η-axis, in the domain 6, the root h_1 also fulfills the inequality $0 < h_1 < h_*$. Therefore, in addition to the pair C_2, C_2', such as before, we have another pair of stationary amplitudes C_1, C_1' (Fig. 2). For these we take the upper sign in (90) and conclude that C_1 is unstable and C_1' stable. Thus in the part of 6 near η-axis we have *two possible states of maintained vibrations*. It depends on the initial disturbance to which of these states the motion will be tending.

III. $|\alpha|$ small, $u\beta < u/(u + \gamma h_*)$ (the boundary of the domains 10, 11 and 7, 8 and its neighbourhood).

We have for $\alpha = 0$:

$$h_1 = 0, \qquad Q_1 > 0, \qquad S_1 \to 0,$$

$h_* < h_2$ when $0 < u\beta < u/(u + \gamma h_*)$, $h_2 < 0$ when $u\beta < 0$.

There are no periodic solutions on the η-axis and in the domains 10, 11 where h_1 becomes negative.

Two stationary amplitudes C_1, C_1' in the domains 7, 8 where $0 < h_1 < h_*$. For small positive α the amplitude C_1 unstable and C_1' stable. Thus in the parts of the domains 7, 8 near η-axis we have *maintained vibrations of large amplitude*. The motion is practically next to divergent.

In conclusion it may be pointed out that it is of interest to follow the variation of the quantities and properties considered in passing from one domain to another.

REFERENCES.

(1) B. V. BULGAKOV, this JOURNAL, Vol. 235, No. 6, pp. 591–616 (1943).
(2) E. V. APPLETON AND B. VAN DER POL, *Phil. Mag.*, Ser. 6, Vol. xliii, pp. 177–193 (1922).
(3) B. VAN DER POL, *Phil. Mag.*, Ser. 6, Vol. xliii, pp. 700–719 (1922).
(4) L. MANDELSTAM AND N. PAPALEXI, "Technical Physics of the USSR," Vol. i, pp. 415–428 (1935).
(5) N. KRYLOFF AND N. BOGOLIÙBOFF, "New Methods of Non-linear Mechanics in Their Application to the Study of the Functioning of the Electron Tube Generators," p. 168, Moscow 1934 (Russian).
(6) R. A. FRAZER, W. J. DUNCAN AND A. R. COLLAR, "Elementary Matrices and some Applications to Dynamics and Differential Equations," Chapt. v, Cambridge, 1938.

(7) "The Engineer," Vol. clxiii, Nos. 4228–4241, pp. 94–95, 122–124, 150–151, 176–177, 201–205, 236–237, 268–269, 294–295, 322–323, 352–353, 380–382, 408–409, 438–439, 467–469 (1937).
(8) W. OPPELT, "Luftfahrtforschung," Bd. 16, Lfg. 8, pp. 447–472 (1939).
(9) V. KOTELNIKOV, "Technica Vosdushnogo Flota," No. 1, pp. 27–31 (1941) (Russian).
(10) N. MINORSKY, *Journ. of Applied Mechanics*, Vol. 9, No. 2, pp. A-65–A-71 (1942).
(11) M. TOLLE, "Regelung der Kraftmaschinen," 3. Aufl., Berlin (1921).
(12) H. K. WEISS, *Journal of Aeronautical Sciences*, Vol. 6, No. 4, pp. 147–152 (1939).
(13) E. L. INCE, "Ordinary Differential Equations," Section 3.23, London, 1927.

AN INTEGRATING RADIATION METER.

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The large number of investigations of artificial nuclear transmutations in progress at the present time bring to the fore problems of the protection of workers from the effects of the high energy radiations involved. Besides taking the usual precautions of avoiding as much as possible exposures to intense radiation and providing adequate shielding, it seems desirable to have some instrument for continuously measuring the radiation level at certain locations, such as the control panels of the equipment, where operators spend a relatively large portion of their time. To be most effective the instrument should be able to integrate the radiation intensity over a whole working day. The simplest and most direct method of measuring the radiation intensity level is to observe directly the ionization produced in a closed vessel with some sort of electroscope. The primary difficulty with this method is to obtain a sufficiently sensitive electroscope with good enough insulation that it loses no appreciable amount of charge in a day. Commercially available instruments, such as the Victoreen meter, are only suitable for much higher radiation intensities. As an alternative, we have designed a rugged, inexpensive radiation meter utilizing a Geiger-Müller counter. The amount of exposure, which is proportional to the total number of counts occurring in the counter, is determined from the amount of gas evolved in a simple electrolytic cell, the current through which is controlled by the counter. This instrument was constructed for use with the cyclotron of the Biochemical Research Foundation at Newark, Delaware, in 1940.

The circuit diagram is shown in Fig. 1. The negative voltage impulse from the counter cuts off the plate current of the first vacuum tube for a time which depends on the magnitude of the counter impulse. The positive voltage from the first tube plate is fed through a small time constant circuit, and turns the second tube on for a short time which is independent of the size of the counter pulse. The plate current of the second tube flows through the electrolytic cell where it releases a given amount of gas per counter impulse. The meter in the plate circuit thus indicates a current proportional to the counting rate while the amount of gas released is proportional to the total number of counts.

The electrolytic cell is sketched in Fig. 2. It was constructed of

pyrex glass with sealed-in tungsten electrodes. The electrodes were tipped with platinum and covered over with glass until only platinum was exposed. The evolved gas collected in the bulb on the left and pushed the liquid up in the capillary tubing on the right. The inside diameter of the capillary was about one millimeter. The stopcock on the left is normally closed, but may be opened to release the evolved gas to adjust the level of the liquid. Distilled water to which was added some sulphuric acid was used as the electrolyte. It is essential to be sure that the liquid is saturated with the evolved gasses before the cell can be relied on for measurements. Switch SW2 in Fig. 1 removes the bias from the second tube in the circuit, and allows current

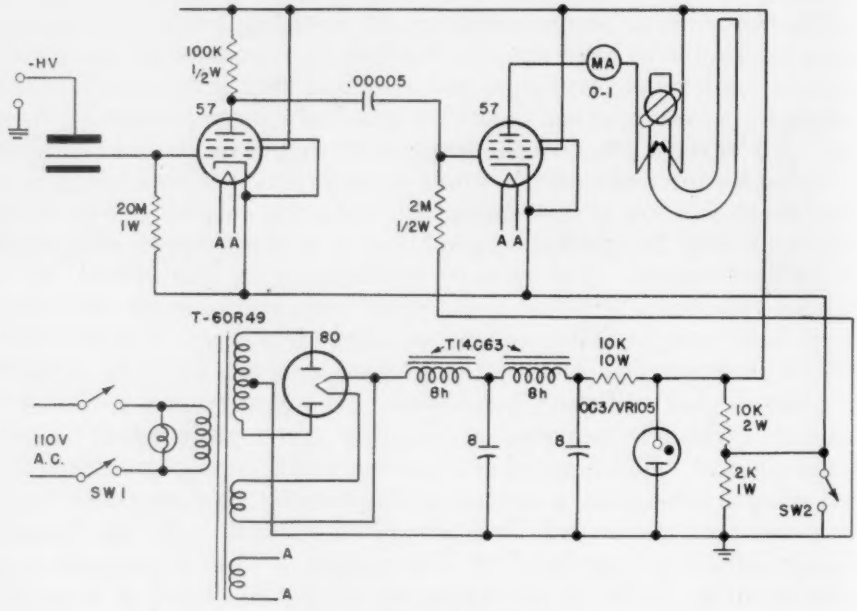

FIG. 1. Circuit Diagram.

to flow through the cell. Gas should be allowed to evolve for several minutes before the zero is set and measurements started.

The counter used had a cathode of copper $\frac{1}{2}$ inch long and an inside diameter of $\frac{1}{8}$ inch with a wall thickness of $\frac{1}{16}$ inch. The wire was a .003 inch tungsten wire, stretched tightly, and spot welded at each end to heavier tungsten which was sealed through the envelope. The envelope was nonex glass about three inches long and $\frac{3}{4}$ inch in diameter. The large volume was intended to prolong the period over which the composition of the gas would not change appreciably under the chemical action of the discharge. The counter was filled with argon to a pressure of 8 cm. of mercury and then 1 cm. of ethyl alcohol vapor added. The

whole counter was dipped in wax containing a red dye to render it insensitive to light. Details of the counter construction and the precautions to be taken have been discussed at length elsewhere.[1] The high potential source for the counter was obtained from a well-regulated supply external to the instrument. The counting rate of this small counter when exposed to 0.1*r* per eight hours was about 12 per second.

The electrical components described above, including the counter, were mounted in a small metal cabinet 7″ × 9″ × 6″ in size. The electrolytic cell was mounted on the front panel, together with the meter and switches.

The radiation meter was calibrated with a standard source of radium. Two milligrams of radium shielded by 1.2 cm. of lead gives a

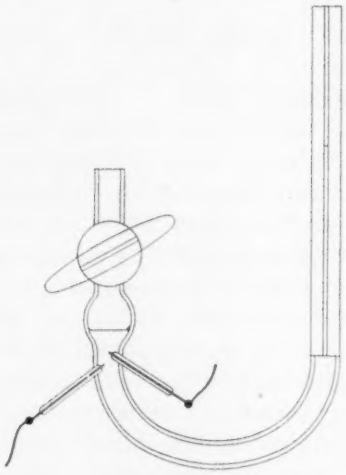

FIG. 2. "Electrolytic Cell."

radiation intensity of 0.1*r* per eight hour day at a distance of 25.2 cm.[2] It was determined with this source that one *r*-unit of exposure corresponded to 92 cm. rise of liquid in the capillary of the electrolytic cell, when a potential of 1,070 volts was applied to the counter. Likewise a full scale reading of the meter, i.e., one milliampere, corresponded to an exposure rate of 10.5*r* units per eight hour day.

One must inquire what is the basis for the assumption that a counter under the conditions of use in this instrument determines something indicative of the effects of an exposure to high intensity radiation. In the first place, the gamma radiation from a cyclotron is not very

[1] C. G. Montgomery and D. D. Montgomery, JOURNAL OF THE FRANKLIN INSTITUTE, 231, 447–467, 509–545, 1941.

[2] "Radium Protection," National Bureau of Standards Handbook H23, U. S. Gov't Printing Office, Wash., 1938.

different in quality from radium radiation, both consisting of rays of energies of the order of a million volts. Second, the number of electrons near the surface of a material is proportional to the absorption coefficient times the range of the electrons in the material. The first of these quantities, if we can neglect pair production, is proportional to the electron density of the material, while the range is inversely proportional to the density. Thus, the product is roughly independent of the material, and the number of electrons appearing on the counter will be proportional to the number of electrons produced inside a living cell. This is not the case if there are appreciable numbers of fast neutrons present, and some other means should be used to determine the danger of exposure to such neutrons. The gamma radiation which accompanies slow neutron capture, however, is recorded in correct proportion to its importance by the counter.

The question of what exposure may be taken with impunity is too complex to be discussed here. A common value for the safe level is 0.1*r*-unit per eight hour day.[2] It may be interesting, however, to show how pessimistic a value for the safe exposure may be obtained. No attempt to justify the value or the reasoning by which it is obtained will be made. Müller and Mott-Smith [3] observed in fruit flies the natural lethal recessive mutation rate to be between 10^{-2} and 10^{-3}, also that an X-ray dosage of 3,420*r* produced a mutation rate of .15. Hence, the dosage necessary to double the natural rate is between 230 and 23*r*-units. If we assume the smaller value and that it applies to man, then over a period of 30 years the maximum exposure should be $2 \times 10^{-3}r$-units per eight hour day. The ionization produced from natural causes (cosmic radiation and normal radioactive impurities) is about $10^{-4}r$-units per eight hours, or about 1/20 of the maximum dosage on this basis.

[3] H. J. Müller and L. M. Mott-Smith, *Proc. Nat. Acad. Sci.*, **16**, 227, 1936.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

LYMAN J. BRIGGS RETIRES, SUCCEEDED BY E. U. CONDON.

Dr. Lyman J. Briggs, Director of the Bureau for the past twelve years, retired on November 5 after a continuous service of over 49 years in the technical work of the Government. Born May 7, 1874, on a farm north of Battle Creek, Michigan, he spent his boyhood in this vicinity. He never attended high school, but was able to enter Michigan State College by examination at the age of fifteen, graduating second in his class four years later. The following two years were spent at the University of Michigan, where he received the degree of Master of Science in physics. The next fall he entered Johns Hopkins University in Baltimore to continue his studies under the renowned Professor Rowland. This was the year that Roentgen announced the discovery of X-rays, and Briggs carried out the first X-ray studies at Johns Hopkins.

In June 1896 he entered the Department of Agriculture as a physicist in its Bureau of Soils, and in 1900 the Secretary of Agriculture, Hon. James Wilson, generously arranged for him to complete his work for his doctorate at Johns Hopkins, without severing his connection with the Department. During his service with the Bureau of Soils, Dr. Briggs originated the centrifugal method of classifying soils, based upon the percentage of water a soil can hold against a force 1,000 times that of gravity. This determination, known as the moisture equivalent, is now widely used in soil laboratories throughout the world.

In 1906 Dr. Briggs was invited to organize the biophysical laboratory of the Bureau of Plant Industry. Here he began his basic studies of the influence of environmental factors on the water requirements of plants, in collaboration with the ecologist, Dr. H. L. Shantz, and during the next 10 years the names of Briggs and Shantz appeared on many monographs in this field. In the course of this work, methods were developed for determining the "wilting coefficient" of a soil, which has found wide application in establishing the amount of water in a soil that is available for plant growth.

On the entry of the United States into the first World War (1917), Dr. Briggs was detailed by executive order to the Bureau of Standards for special work on a stable zenith instrument for the Navy. In 1920 he became a member of the Bureau's staff and chief of the Mechanics and Sound Division. Here he was associated with Dr. Paul R. Heyl,

* Communicated by the Director,

and together they invented the earth inductor compass for which they received the Magellan Medal in 1922. In 1926 Dr. Briggs was appointed Assistant Director for Research and Testing, under Dr. George K. Burgess. On the sudden death of the latter in July 1932, Dr. Briggs was selected for the vacancy by President Hoover, but the appointment, along with others, was not confirmed. When Mr. Roosevelt became President he immediately resubmitted Dr. Briggs' name, which was approved by the Senate. He thus has the distinction of having received his appointment from two Presidents.

At the time Dr. Briggs became Director, the country was in the depths of the depression, from which the Bureau suffered, along with other organizations and individuals. Reduced appropriations made necessary a severe curtailment in the staff and in the work that could be carried on, particularly in basic research. Dr. Briggs handled this difficult and discouraging situation with such judgment and fairness that he won the respect of every Bureau employee. On June 30, 1934, the staff numbered 668, the lowest in many years. As business conditions improved, many of those who had been laid off were reinstated and new appointments were made, but even on June 30, 1940, the staff was smaller than it had been 8 years before.

Under pressure of preparedness and war, the number of Bureau employees was rapidly increased, until on June 30, 1944, the total was 2,279. The usual difficulties of recruitment were aggravated by the war, including not only the demands for men by the armed services, but the competition of private industry and of newly created agencies within the Government with their higher pay scales and more rapid advancement. As military problems were submitted in increasing volume, the Bureau was placed on a war footing, its grounds became a prohibited zone, and by 1942 the Director estimated that 90 per cent of the personnel and equipment were engaged in war work. As a scientific laboratory, the Bureau had always been a place where the greatest freedom of thought and of discussion prevailed. For security reasons, this had to be changed, almost over night. That this was accomplished without significant loss of esprit de corps is due largely to the good sense and example of the Director.

Although Dr. Briggs' record as an administrator is an enviable one, he considers himself to be, first of all, a physicist and is never so happy as when in the laboratory. Perhaps it was for this reason that he was selected by the President in 1939 as chairman of the original Uranium Committee to study the possibility of using atomic energy in warfare. His connection with this project was unknown to most of the Bureau's staff, yet he was closely identified with all the earlier work and personally directed much of the research.

Dr. Briggs has been a member of the National Advisory Committee for Aeronautics since 1933 and was elected vice chairman in 1942. He

has been a member of the aerodynamics subcommittee for 23 years, and during the time that he headed the Bureau's Mechanics and Sound Division he gave particular attention to its researches in air flow. With Dr. Hugh L. Dryden he made pioneer measurements of flow around airfoils at very high speeds up to and exceeding the speed of sound. The results of this work found application in determining the blade form of aircraft propellers. He has always enjoyed a difficult problem in the design of apparatus and was selected to head the instrument work in connection with the two stratosphere balloon flights sponsored by the National Geographic Society and the Army Air Service. Typical examples at the Bureau are his machines for floating metallic fatigue specimens on a layer of compressed air, and for measuring the forces on an elastic ball when struck. His interest in the theoretical side of his work is equally keen. He accepted the sponsorship of the "Project for the Computation of Mathematical Tables" (Work Projects Administration for the City of New York) in 1938. In all, 57 tables have thus far been made available to the public; the project is being continued with the support of the Office of Research and Inventions of the Navy Department.

Dr. Briggs became chairman of the Federal Specifications Board in 1932, of the Federal Fire Council in 1933, and of the National Conference on Weights and Measures in 1935. He is a life trustee of the National Geographic Society, and has served as chairman of its Research Committee. He is a director of the American Standards Association, and a member of the American Physical Society, Washington Academy of Sciences, Philosophical Society of Washington, American Philosophical Society, Tau Beta Pi, and Sigma Xi. He is a member and past president of the Cosmos Club and of the Federal Club. He was elected a fellow of the National Academy of Sciences in 1942. In 1932, his alma mater, Michigan State College, conferred on him the honorary degree of Doctor of Science, to be followed by an LL.D. from the University of Michigan in 1936. He has been honored by degrees of Doctor of Engineering from the South Dakota School of Mines in 1935, and Doctor of Science from George Washington, Georgetown, and Columbia Universities.

Dr. Briggs was married to Katharine E. Cook of Clarement, California, in 1896. They had two children, Albert Cook, who died in 1901, and Isabel (now Mrs. C. G. Myers). A grandson, Peter Myers, is serving in the United States Naval Reserve. The facilities of the Bureau's laboratories have been offered to Dr. Briggs as "Director Emeritus of the National Bureau of Standards" by Secretary Wallace. His associates hope that he may be willing to resume some of the research that he was obliged to drop when he became Director.

Dr. E. U. Condon, who succeeds Dr. Briggs, was born March 2, 1902, in Alamogordo, New Mexico. He was graduated from the Uni-

versity of California in 1924, and received his doctor's degree from that institution in 1916. During 1926–1927 he was a National Research Fellow at Göttingen and Munich, and was a physics lecturer at Columbia University in 1928. He served as assistant professor of physics at Princeton during 1928–1929 and as associate professor from 1930 till 1937. During 1929–1930 he was professor of theoretical physics at the University of Minnesota, and since 1937, advisory professor at the University of Pittsburgh. He became associate director of Westinghouse Research Laboratories in 1937, a position which he occupied till his appointment by President Truman to the Directorship of the Bureau.

Dr. Condon served with Dr. Briggs on the original Uranium Committee and had charge of many top secret projects during the war. He is an internationally recognized authority in the new field of electronics and is co-author of treatises on quantum mechanics and theory of atomic spectra. He is a fellow of the National Academy of Sciences, and is now serving as scientific adviser to the special committee on atomic energy of the United States Senate.

INTERSERVICE RADIO PROPAGATION LABORATORY.

The important part played by the Bureau during the war in predicting radio transmission conditions between any two places on the earth's surface, has just now become known to the public through the release of this work from a "classified" status.

Early in the war, a group of experts, working under the general supervision of Dr. J. H. Dellinger and Dr. Newbern Smith, were organized as the United States centralizing group for ionospheric and radio propagation data taken all over the world. They were designated as the IRPL (Interservice Radio Propagation Laboratory) and served under the Wave Propagation Committee of the Joint Communications Board, which, in turn, came under the U. S. Joint Chiefs of Staff.

The primary function of the IRPL was to furnish radio propagation information and predictions to the Allied Military Services and to advise them on radio communication problems of operational importance. This information was particularly needed because of the widespread use of radio in the far corners of the world, the necessity for efficient utilization of frequencies in the crowded radio spectrum, and the importance of knowing just where radio waves were going and how to insure their arriving at their destination. To this end the laboratory prepared monthly world-wide charts of predicted ionospheric data, from which usable frequencies for any path could be calculated; distributed all available ionospheric and radio propagation data to the

armed services, commercial users, and scientific laboratories; sent out warnings of expected radio disturbances associated with "storms" in the ionosphere; carried on research in radio wave propagation and in the techniques of prediction; developed methods for solving high-frequency radio propagation problems; studied radio wave absorption in the ionosphere; analyzed radio traffic data from the propagation viewpoint; correlated high-frequency direction-finder errors with ionosphere conditions; and, as an essential part of the entire program, operated the Bureau's field station at Sterling, Va., where observations were made and new apparatus developed. A considerable part of this work is being continued because it has important peacetime applications.

One of the important jobs was the training of military radio men in the operational use, both tactical and strategical, of radio propagation data and predictions. This was particularly needed because of the hitherto limited knowledge and application of the information, since there were, at the beginning of the war, a very few men who had had any background or training in this field. In this connection the laboratory issued the "IRPL Radio Propagation Handbook," which gave the essential facts of radio propagation and rapid practical methods for solution of radio propagation problems.

Probably the most valuable information made available by the group appears in its IRPL–D series of monthly reports, entitled "Basic Radio Propagation Predictions for (date)—Three Months in Advance." Thus IRPL–D15, issued on November 1, gave predictions for February, 1946. Another regular monthly report constitutes the IRPL–F series, "Ionospheric Data." These give graphs and tabulations of vertical-incidence ionosphere characteristics for all the ionosphere stations in the world from which data are obtainable. Other reports, giving the results of research developments, are issued in an IRPL–R series. A recent one, IRPL–R23, is an "unclassified" (no restrictions on distribution) report, entitled "Solar Cycle Data for Correlation with Radio Propagation Phenomena."

Associated directly with the IRPL are laboratories at Stanford University, California; Louisiana State University, Baton Rouge, La.; University of Puerto Rico, San Juan, P. R.; Harvard University, and Massachusetts Institute of Technology, both at Boston, Mass. Valuable help has been received from the Carnegie Institution of Washington, Department of Terrestrial Magnetism, which maintained observatories at the University of Alaska and other places, and also centralized solar and magnetic data. Ionospheric and radio propagation data are centralized by the National Bureau of Standards from about 50 stations operated by the United States, Great Britain, Canada, Australia, New Zealand, and the U. S. S. R. These world-wide sources of information are listed each month in the IRPL–F series of reports.

DISSOCIATION CONSTANTS CALCULATED FROM ULTRAVIOLET SPECTRA.

Most organic compounds that are colorless in the visible region of the spectrum, have absorption bands in the ultraviolet. Spectrophotometric measurements in the ultraviolet ranges have now been obtained with such accuracy that the dissociation constants of weak electrolytes can be calculated. In the December number of the *Journal of Research* (RP1686), Elizabeth E. Sager, Marjorie R. Schooley, Alice S. Carr, and S. F. Acree report results on solutions of *p*-hydroxybenzoic acid, several of its esters, and potassium *p*-phenolsulfonate at 25° C. by this method.

The absorption bands of the ionic and the molecular forms of each compound were first obtained, with the use of acid, alkali, or buffers of widely different pH values. Several intermediate steps in the transformation of one form into the other were then measured. The pH of each solution was controlled by buffers of low ionic strengths, of known pH values, at each step in the transformation. From the spectral transmittancies at one or several wave-lengths, the amounts of the molecular and ionic forms can be calculated. When the pH of the buffered solution is known, the apparent dissociation constant, or pK^*, can then be determined.

The pK^* values of methyl, ethyl, *n*-butyl, and benzyl *p*-hydroxybenzoate are all approximately 8.3. The pK^* values for the first and second steps in the dissociation of *p*-hydroxybenzoic acid were found to be 4.5 and 9.3. The estimated values of pK_a, the negative logarithm of the thermodynamic dissociation constant, for potassium *p*-phenolsulfonate is 9.03. This value is 0.02 of a unit lower than the value found by e.m.f. studies.

REACTION OF PERIODIC ACID ON THE DIFRUCTOSE ANHYDRIDES.

As set forth in a report by Emma J. McDonald and the late Richard F. Jackson in the December *Journal of Research* (RP1683), the difructose anhydrides have been reacted with periodic acid. In keeping with their known structures, difructose anhydrides III and I react, respectively, with one and two moles of acid. Difructose anhydride II reacts with one mole of periodic acid; thus it contains only two adjacent carbons carrying free hydroxyl groups. From this fact, along with previously determined data, it is suggested that 1,2′-2,4′-difructofuranose is the most probable structure for difructose anhydride II.

THERMODYNAMIC PROPERTIES OF ACETYLENE HYDROCARBONS.

The December number of the *Journal of Research* contains a report (RP1682) entitled, "Heats, Equilibrium Constants, and Free Energies of Formation of the Acetylene Hydrocarbons through the Pentynes, to

1500° K." by Donald D. Wagman, John E. Kilpatrick, Kenneth S. Pitzer, and Frederick D. Rossini. The report, prepared as part of the work of the American Petroleum Institute Research Project 44 and the Bureau's thermochemical laboratory, presents values for the following thermodynamic properties, for acetylene, propyne (methylacetylene), 1-butyne (ethylacetylene), 2-butyne (dimethylacetylene), 1-pentyne (*n*-propylacetylene), 2-pentyne (methylethylacetylene), and 3-methyl-1-butyne (isopropylacetylene), in the gaseous state, to 1500° K. The heat content function, $(H^\circ - H^\circ_0)/T$; the free energy function, $(F^\circ - H^\circ_0)/T$; the entropy, S°; the heat content, $H^\circ - H^\circ_0$; the heat capacity, C_p°; the heat of formation from the elements, ΔHf°; the free energy of formation from the elements, ΔFf°; and the logarithm of the equilibrium constant of formation from the elements, $\log_{10} Kf$.

Equilibrium constants and concentrations of components are given in tabular and graphical form for the isomerization of the two butynes and of the three pentynes as a function of temperature to 1500° K. Equilibrium constants are also given in tabular and graphical form for the dehydrogenation of ethane to ethylene to acetylene, of propane to propylene to propyne, of *n*-butane to 1-butene to 1-butyne, and of *n*-pentane to 1-pentane to 1-pentyne.

DENSITY BALANCE FOR LIQUID HYDROCARBONS.

Research Paper RP1685 in the December number of the *Journal of Research* deals with the assembly and calibration of a density balance for liquid hydrocarbons. This is a development by Alphonse F. Forziati, Beveridge J. Mair, and Frederick D. Rossini, of the American Petroleum Institute Research Project 6 in the Bureau's Section on Thermochemistry and Hydrocarbons. With this apparatus, samples as small as 9 ml. in volume may be measured with a reproducibility of ± 0.00003 g./ml. and an estimated overall uncertainty of ± 0.00005 g./ml.

Long Range Weather Probabilities.—More accurate "know-when" of troublesome drought, as well as other weather hazards, may be added to the farmer's growing fund of weather knowledge as the result of a study made by the U. S. Department of Agriculture, from Weather Bureau records.

The Soil Conservation Service, which made the study, has developed a statistical method of gaging the probable occurrence of drought in any locality in the United States at any time of the year. Although the complex method is necessarily limited to use by scientists, the information obtained may be used by agronomists and others for the farmer's benefit, in adapting soil and water conservation work, as well as other farm activities, to weather conditions. It is possible that in time a farmer (or any other person) might be given a chart showing the probability of weather hazards in his locality, for virtually every day of the year.

Because the information obtained shows when sequences of dry or rainy days are most likely to occur, it can be valuable in checking day to day weather forecasts and in long range planning as well. It can be used, for instance, in planning terrace construction programs for periods when rain is least likely to cause erosion of unfinished embankments; or in extensive seedings of grass, for selecting a planting time when a killing drought is least apt to occur during the period needed for germination of the seed. Agricultural workers probably will evolve many other practical uses for the material provided by the study.

Applying to drought the theory of probability used by insurance authorities in deriving life expectancy tables, the SCS research divison puts into the hands of climatologists and other scientists a technique that also can be employed to discover probable occurrences of other climatic hazards such as intensities of rainfall and extremes of temperature. Drought was selected for trying out this method because drought data already has been compiled and tabulated, from Weather Bureau records of 1898 to 1937, for stations representing every climatic area of the continental United States.

R. H. O.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING.

The Franklin Institute held a stated monthly meeting at 8:15 on Wednesday, December 19, 1945, in the Hall of the Institute. Commander W. Chattin Wetherill, Vice-President, presided.

The meeting was called to order after the playing of the National Anthem.

Upon motion, the minutes of the October meeting were approved as printed in the November JOURNAL.

Dr. Henry Butler Allen, Secretary, reported on membership as follows:

Sustaining	1
Active	41
Associate	32
Student	26
	100
Total membership as of December 1, 1945	5,468

He then spoke of the Christmas Week Lectures for Young People, given on the James Mapes Dodge Foundation, and suggested the idea of tickets as Christmas gifts. He announced the lecturer would be Faber Birren, noted consultant on color, and that his subject would be, "The World of Color."

As indicated in the By-Laws, Article IV, Section 4, nomination of officers and certain members of the Board of Managers takes place at the December meeting of The Franklin Institute.

Accordingly, Dr. Allen read the following nominations, and stated that they had been submitted by the required number of members, and in writing, as subscribed to in the By-Laws:

President (to serve one year)	CHARLES S. REDDING
Vice-Presidents (to serve one year)	WALTON FORSTALL
	W. CHATTIN WETHERILL
	SAMUEL S. FELS
	RICHARD W. LLOYD
	RICHARD T. NALLE
Treasurer (to serve one year)	MORTON M. PRICE
Members of the Board of Managers (to serve three years)	IRENEE DUPONT
	RUPEN EKSERGIAN
	W. H. FULWEILER
	HORACE P. LIVERSIDGE
	MALCOLM LLOYD, JR.
	MARSHALL S. MORGAN
	EDWARD WARWICK
	D. ROBERT YARNALL

The presiding officer called for nominations from the floor.

As none were offered, upon motion, duly seconded and carried, the nominations were declared closed.

Commander Wetherill then introduced the speaker of the evening, Dr. Robert Strausz-Hupé, Lecturer in Political Science, University of Pennsylvania, who spoke on "National Resources and Foreign Policy."

Dr. Strausz-Hupé pointed out that the world political responsibilities of the United States have increased immensely as a result of the recent war, and stated that it is the sum total of national resources, spiritual, intellectual and material, which determines a nation's role in world affairs. He discussed these matters, impressing upon his audience the fact that the American people must be made aware of the nature of the foriegn policy this country should assume.

The meeting was dismissed with a rising vote of thanks to the lecturer for his stimulating address.

HENRY BUTLER ALLEN,
Secretary.

NEW MEMBERS OF THE FRANKLIN INSTITUTE ELECTED, JUNE 1, 1945, TO DECEMBER 1, 1945.

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Mr. Charles E. Ebert

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COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, December 12, 1945.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 12, 1945.

MR. JAMES D. LEE, JR. *in the chair.*

The following report was presented for final action:

No. 3110: Work of Sanford A. Moss.

This report recommended the award of a Howard N. Potts Medal to Sanford A. Moss, of Lynn, Massachusetts, "In consideration of the extreme value of his work in making a turbo-supercharger a successful and reliable part of an internal combustion aeronautical engine."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. to 5 o'clock P.M., Wednesdays and Thursdays from two until 10 o'clock P.M.

RECENT ADDITIONS.

BUILDING.

MOLLOY, E., Editor. Hot-Water Engineering. 1945.

BUSINESS.

Alexander Hamilton Institute. Modern Business. Twenty-seven Volumes. 1929–1930.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BELL, H. S. American Petroleum Refining. Third Edition. 1945.

Chemical Rubber Publishing Company. Handbook of Chemistry and Physics. Twenty-Ninth Edition. 1945.

DEMENT, JACK. Fluorochemistry. 1945.

DEMENT, JACK, AND H. C. DAKE. Uranium and Atomic Power, with Appendix on the Atomic Bomb. 1945.

Hopkin and Williams. Staff of the Research Laboratory. Organic Reagents for Organic Analysis. 1944.

LECAT, MAURICE. L'Azéotropie. Tome Second. 1942.

LEDOUX, EDWARD. Vapor Adsorption. 1945.

MONCRIEFF, R. W. The Chemical Senses. 1944.

EDUCATION.

Stevens Institute of Technology. Alumni Association. Stevens 75th Anniversary. 1945.

ELECTRIC ENGINEERING.

AITKEN, WILLIAM. Who Invented the Telephone? 1939.

DEWEESE, FRED C. Transmission Lines. First Edition. 1945.

EDDY, WILLIAM C. Television. 1945.

HUBBELL, RICHARD. Television: Programming and Production. 1945.

KRAMER, ANDREW W. Elementary Engineering Electronics. First Edition. 1945.

MACCOLL, LEROY A. Fundamental Theory of Servomechanisms. 1945.

PRATT, W. T. Worked Examples in Electrotechnology. No date.

ENGINEERING.

Society for Experimental Stress Analysis. Proceedings. Volume 3, Number 1. 1945.

TIMOSHENKO, S., AND D. H. YOUNG. Theory of Structures. First Edition. 1945.

MATHEMATICS.

HOOPER, A. The River Mathematics. 1945.

MANLEY, R. G. Wave Form Analysis. 1945.

MILNE-THOMSON, L. M. Calculus of Finite Differences. 1933.

STERNBERG, WOLFGANG J., AND TURNER L. SMITH. The Theory of Potential and Spherical Harmonics. 1944.

TURNBULL, H. W. The Mathematical Discoveries of Newton. 1945.

MECHANICAL ENGINEERING.

COLVIN, FRED H., AND FRANK A. STANLEY. American Machinists' Handbook. Eighth Edition. 1945.

PALMGREN, ARVID. Ball and Roller Bearing Engineering. 1945.

TREACY, JOHN. Production Illustration. 1945.

MINING AND METALLURGY.

LIDDELL, DONALD M., Editor. Handbook of Nonferrous Metallurgy. Volume 2: Recovery of the Metals. 1945.

ROWLANDS, DOROTHY HOWARD. Coal—And All About It. 1945.

TAYLOR, A. An Introduction to X-Ray Metallography. 1945.
WOLDMAN, NORMAN E., AND ROGER J. METZLER. Engineering Alloys. 1945.

NAVAL ARCHITECTURE.

Society of Naval Architects and Marine Engineers. Historical Transactions 1893–1943. 1945.

PHYSICS.

ADDISON, HERBERT. A Treatise on Applied Hydraulics. Third Edition. 1944.
ROBERTSON, JOHN KELLOCK. Atomic Artillery and the Atomic Bomb. 1945.
WOOD, A. B. A Textbook of Sound. Second Edition. 1944.

RAILROADS.

ROBERTSON, ARCHIE. Slow Train to Yesterday. 1945.

SCIENCE.

BAKER, JOHN R. Science and the Planned State. 1945.

TEXTILES.

BRADBURY, FRED. Calculations in Yarns and Fabrics. No date.
BRADBURY, FRED. Worsted Preparing and Spinning. Volumes 1–3. No date.
LEGGETT, WILLIAM F. The Story of Linen. 1945.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

A Filtration Error in Nitrogen Determinations.—JAMES L. LEITCH AND LORNA A. WELLS. In a study of the protein fractions of aqueous extracts of various organs and particularly of bone marrow, satisfactory clarification was only obtained by repeated filtration through Schleicher and Schuell filter paper No. 576. When the extract, after each successive filtration, was analyzed for total nitrogen and for non-protein nitrogen after trichloroacetic acid precipitation, it was found that both values increased with each filtration. Calculations from these data showed that the protein concentration of the extract varied irregularly while the corresponding values for the non-protein nitrogen increased with each filtration. Consequently, it was not possible to obtain any accurate value for the protein concentration of this extract or of any others similarly prepared.

Before further work could be carried out on these organ extracts, some explanation or correction for these variations was necessary. It seemed logical to conclude that the variations in nitrogen content were due, in some manner, to the filtration procedure and probably to soluble nitrogen-containing constituents of the paper. If this were the case, some other means of filtration might be required. The work herein reported was, therefore, undertaken to determine quantitatively the effects of filtration through various types of filter paper and also through special sintered glass filters on the total and non-protein nitrogen values of protein solutions. No originality is claimed for the small sintered glass filters described herein nor for reporting the filtration error in nitrogen determinations since it may be assumed that these have been known previously. However, the absence of references thereto in the pertinent literature justifies the publication of this brief note.

PREPARATION OF SPECIAL SINTERED GLASS FILTERS.

Small sintered glass filters were prepared easily and inexpensively as follows: Pyrex glass was ground up, washed, and graded through 60, 80, 100 and 120-mesh screens. For the present work, powdered glass was used which passed a 100-mesh screen but was retained on a 120-mesh. Pyrex tubes of 8 mm. inside diameter and about 5 cm. long were used for the body of the filter. A carbon rod (old arc carbon or drycell carbon), just fitting inside the tube, was inserted three-fourths of the way into the glass tube (Fig. 1a) and sufficient powdered glass of the appropriate mesh size was placed in the tube so as to form a layer approximately 2–3 mm. thick on the rod. The powdered glass was tamped down and held in place by the insertion of a second carbon rod,

the filter heated for about one minute in an oxygen-gas flame, one carbon rod withdrawn from contact with the glass disc and the filter again heated until the entire sintered glass disc assumed a dull cherry-red color. The filter was then slowly annealed in the yellow flame. A short length of pyrex tubing (2–3 mm. inside diameter) was finally fused to the bottom of the filter to give the completed unit (Fig. 1b).

The dimensions of the sintered glass filter described above may be modified to suit the purpose for which it is to be used. However, filters larger than 15 mm. in diameter cannot be made satisfactorily by this procedure because of the difficulties in heating these larger discs uniformly without fusing some of the powdered glass.

Since some variations in porosity must be expected in filters prepared in the above manner, a layer of filter-aid, previously thoroughly washed with distilled water, was deposited on the glass disc just prior to its use. For the filters employed in the present work, the layer of filter-aid was

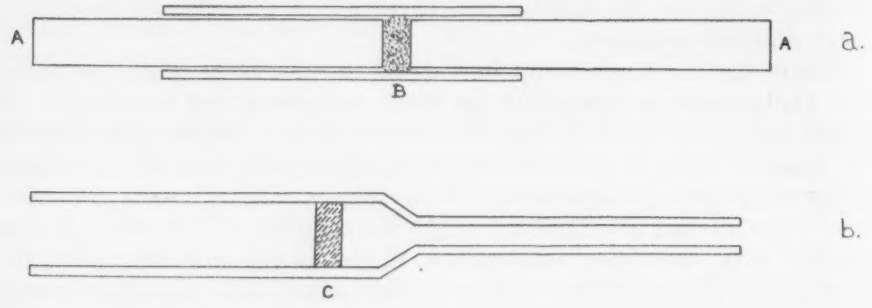

FIG. 1. Sintered glass filter. a. Filter prior to fusing of powdered glass. b. Completed filter. A—Carbon rod. B—Powdered glass. C—Sintered glass disc.

formed by filtering 5 ml. of a 1 per cent. suspension of Johns-Manville's Standard Supercel. In this manner, a standard and reproducible filtering surface was available at all times. Furthermore, the use of filter-aids is advantageous in that it prevents clogging of the glass disc by retained proteins and renders the filter easily cleaned by a reverse flow of water through the disc. A small amount of the dry Standard Supercel may also be added to the solution being filtered to speed up the rate of filtration. The use of filter-aids does not affect the results of the nitrogen determinations.

FILTRATION OF BLOOD-PROTEIN SOLUTIONS.

To determine the order of magnitude of the increase in the nitrogen values produced by filtration through Schleicher and Schuell filter paper No. 576 and to compare this result with comparable data obtained by using the sintered glass filters described above, a 25 ml.-sample of a blood-protein solution was filtered five times by each

method. Samples were removed prior to the first filtration for the determination of total and non-protein nitrogen by the method of Ma and Zuazaga (1) and similarly after each successive filtration. The non-protein nitrogen was determined after precipitation by trichloroacetic acid and filtration through the same type of filter as was used for the corresponding protein solutions. The results of these two experiments are summarized in Table I.

TABLE I.

Comparison of the Effects of Successive Filtrations on the Protein and Non-protein Nitrogen Concentrations of Two Blood Protein Solutions.

Number of Filtrations	Sample Filtered Through Schleicher and Schuell No. 576 Filter Paper		Samples Filtered Through Small Sintered Glass Filter	
	Protein mg./ml.	N.P.N. mg./ml.	Protein mg./ml.	N.P.N. mg./ml.
0	2.310	0.116	2.539	0.082
1	2.125	0.135	2.456	0.078
2	2.132	0.146	2.469	0.075
3	2.180	0.142	2.442	0.082
4	2.176	0.153	2.446	0.078
5	2.095	0.174	2.433	0.082

Similar results were obtained with Whatman filter papers Nos. 1, 40, 42 and 43, although for these papers the variations in protein concentrations were slight and the increase in the non-protein nitrogen smaller than those given in Table I.

NITROGEN BLANKS FROM FILTER PAPER.

The data in Table I led to the conclusion that some nitrogenous compound, possibly ammonium salts, was being eluted from the filter paper by the protein solutions and especially by the trichloroacetic acid solutions used for non-protein nitrogen determinations. To substantiate the above conclusion, various types of filter paper from freshly opened packages were used for the filtration of 10 ml. volumes of distilled water and the resulting filtrates were analyzed for total nitrogen. For some of the filtrates one 2 ml.-aliquot was analyzed without the usual sulfuric-acid digestion and a second after digestion with sulfuric acid. The results of these experiments in terms of the volume of N/100 hydrochloric acid used for titration of the distillates are summarized in Table II. Similar results were obtained when trichloroacetic acid was substituted for the distilled water in these tests.

DISCUSSION.

Since the data given in Tables I and II indicate that a definite error is introduced into protein and non-protien nitrogen determinations preceded by repeated filtration through filter paper, a general survey of the pertinent literature was made. The only reference to a correction

TABLE II.

Nitrogen Blanks for Various Filtration Procedures.
(Data reported as milliliters of N/100 HCl)

Treatment	Blank Undigested ml.	Blank Digested ml.	Difference Between Unfiltered and Filtered Blanks	
			Undigested ml.	Digested ml.
None	0.08	0.10	—	—
Filtered through Whatman No. 1	0.10	0.12	0.02	0.02
Filtered through Whatman No. 40	0.12	0.15	0.04	0.05
	—	0.10[1]	—	0.00
Filtered through Whatman No. 42	0.12	0.13	0.04	0.03
Filtered through Whatman No. 43	—	0.11	—	0.01
Filtered through Schleicher and Schuell No. 576	0.13	0.18	0.05	0.08
Filtered through sintered glass filter	0.08	0.10	0.00	0.00

[1] Filter paper previously washed with distilled water.

for nitrogen in filter paper was found in Hawk and Bergeim (2). These authors, in discussing the Folin gravimetric method for the determination of protein in urine, state in a footnote "Correction should be made for the nitrogen content of the filter paper unless this factor is negligible." For the determination of non-protein nitrogen, Bray (3), Hawk and Bergeim (4), Peters and Van Slyke (5) and Stitt, Clough and Clough (6) recommend filtration without any qualification except that the last-named authors recommend ammonia-free filter paper. In the original description for the preparation of Folin-Wu blood filtrates (7), no statement is made regarding the type of paper used or the possibility of a correction being required for nitrogen in the paper.*

From the data in Table I, it can readily be seen that the non-protein nitrogen values for protein solutions after repeated filtration through filter paper may be in error by as much as 100 per cent. while the corresponding data for the protein concentrations vary irregularly. On the other hand, comparable data obtained using the special sintered glass filters described herein are constant within the limits of the analytical method (± 0.01 ml. of N/100 hydrochloric acid in the final titration). The initial drop in the protein concentration after a single filtration through sintered glass filters represents a removal of protein which was also indicated by the greater clarity of the initial filtrate.

It was first thought that some correction could be applied for the

* The use of centrifugation in non-protein nitrogen determinations is suggested by Peters and Van Slyke (5). This procedure is not discussed in the present paper since centrifugation was not found suitable for large numbers of determinations and, furthermore, centrifuged extracts not as satisfactorily clarified as those prepared by filtration.

nitrogen in filter paper. When the nitrogen blanks, reported in Table II, had been obtained, the values were relatively constant for the same type of filter paper indicating that a correction might be possible. However, correction of the data given in Table I for the blank for the Schleicher and Schuell filter paper No. 576 does not result in constant values. It is apparent that the amount of nitrogen eluted from the filter paper will depend on the volume of solution filtered, on the time required for filtration, and quite probably on the nature (pH, salt content, etc.) of the solution. In view of all these probable factors, it is not surprising that the data cannot be corrected.

The general agreement between the values for extracted nitrogen determined with and without digestion as given in Table II suggest that most of the nitrogen occurs in the filter paper as ammonia or ammonium salts. Washing of the papers † with distilled water prior to use would be one procedure for eliminating this source of error. Such a procedure would be time-consuming and would hardly be practical when large numbers of determinations were involved. However, the use of sintered glass filters, as previously described, entirely eliminates the need for any correction for nitrogen and leads to a procedure which is very rapid.

The error due to the presence of nitrogen in filter paper will be quite appreciable when repeated filtrations of aqueous protein solutions are involved. However, it may also be significant in single filtrations if a paper of relatively high nitrogen content is used, leading to errors in excess of 50 per cent.

Summary. Filtration through standard filter papers may introduce a serious error in total and non-protein nitrogen determinations particularly when the procedure involves repeated filtration prior to analysis. Evidence is presented which indicates that some, if not all, of the soluble nitrogen in filter paper is present as ammonia or ammonium salts. This error can be completely eliminated by the use of small sintered glass filters.

REFERENCES.

(1) MA, T. S., AND ZUAZAGA, T., *Ind. Eng. Chem., Anal. Ed.*, **14**, 280, 1942.
(2) HAWK, P. B., AND BERGEIM, O., "Practical Physiological Chemistry," 11th Ed., P. Blakiston's Son and Company, Philadelphia, 1937, p. 744.
(3) BRAY, W. E., "Synopsis of Clinical Laboratory Methods," 3rd Ed., C. V. Mosby Company, St. Louis, 1944, pp. 64 and 197.
(4) HAWK, P. B., AND BERGEIM, O., *loc. cit.*, p. 417.
(5) PETERS, J. P., AND VAN SLYKE, D. D., "Quantitative Clinical Chemistry," Volume II, "Methods," Williams and Wilkins Company, Baltimore, 1932, pp. 534–5.
(6) STITT, E. R., CLOUGH, P. W., AND CLOUGH, M. C., "Practical Bacteriology, Haematology and Animal Parasitology," 9th Ed., The Blakiston Company, Philadelphia, 1938, p. 650.
(7) FOLIN, O., AND WU, H., *J. Biol. Chem.*, **38**, 81, 1919.

† Since the completion of this work, George A. Neville, CPhM, USN, in a personal communication informed the senior author that prewashing and oven drying of filter papers has were been a recommended procedure in the Medical Department of the United States Navy.

BOOK REVIEWS.

SYMPOSIUM ON QUARTA OSCILLATOR-PLATES. Reprinted from American Mineralogist, Volume 30, 1945. 264 pages, illustrations, 16 × 23 cms.

This collection of papers represents a summary of considerable breadth of knowledge of the subject. World War II has caused great demands on the quartz crystal industry both from the production as well as from the development points of view, so this summary is of added importance as a recital of present day position.

There are thirteen papers by five authors in the symposium. The first, by Clifford Frondel gives a history of the quartz oscillator plate industry with special emphasis on the period 1941 to 1944. Following this is a paper entitled "The Piezoelectric Quartz Resonator" by Karl S. Van Dyke wherein are described the properties of quartz, the mechanism of current influence by quartz, the importance of constant frequencies in communication and of the orientation of quartz radio elements. It is predicted in this paper that the powers of quartz in new fields of more extreme frequency precision or into higher frequency ranges have been far from being exploited. The paper following this on the geology of quartz deposites gives information gathered during more than three years work by each of the three authors. The inspection and grading of quartz is the heading of a paper describing the defects of the crystal and their detection, and another paper along the same lines treats on the effect of imperfections on the usability of quartz for oscillator plates.

The theory, methods and equipment used in orienting raw quartz and the reorientation of sections, bars, wafers and blanks in various stages of processing are described in the first of a series of papers on quartz-working. Others of the papers include quartz cutting, sawing and machine lapping of quartz oscillator plates. Subsequently, space is given to final frequency adjustment, effect of radiation on elasticity, and secondary Dauphiné twinning of quartz. Finally there is a glossary of terms used in the industry.

These papers, all in this single binding, are reprinted from *American Mineralogist*, 30: 205–468 (1945).

R. H. OPPERMANN.

BIOENERGETICS AND GROWTH WITH SPECIAL REFERENCE TO THE EFFICIENCY COMPLEX IN DOMESTIC ANIMALS, by Samuel Brody. 1023 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1945. Price $8.50.

This book reveals a study of the energetic efficiencies of agricultural processes such as those concerned in the production of meat, milk, eggs, and muscular work along with the factors influencing these efficiencies. The animal husbandman, by employing special techniques, finds it possible to accelerate the speed of growth, milk and egg production etc., of his animals. Whether this increased productivity results in greater immediate energetic efficiency of course depends on the relative acceleration of the productive processes on the one hand and the maintenance cost on the other. Also the effect on future health, fertility, and longevity must be considered. This problem is many sided, and this book presents an integration of the results of the researches on the subject sponsored by the Herman Frash Foundation for Research in Agricultural Chemistry at the Missouri Agricultural Experiment Station.

The twenty-four chapters of the work represent a break-up of the efficiency complex into its several components. After a brief introduction there is disscused energetics and energy units, the plane of nutrition, metabolic catalysis, homeostasis and organismic theory homeothermy, basal energy, maintenance needs, growth and ageing, and monetary economics. At the end there are subject and author indexes of the great variety of material given in the text.

Many of the problems discussed have not yet been investigated. The theoretical feature of the book is the attempt to generalize large bodies of cumbersome data by simple rational but

broadly inclusive equations. Much practical information is brought to light as, for example, how metabolism and maintenance costs vary with age and weight, or how temperature influences productive processes, or how body size of cattle influences energetic efficiency and profit on milk production. The coverage of the book is over a wide territory and involved reports and papers from many publications. The book is exceedingly valuable in its field.

R. H. OPPERMANN.

TABLE OF ARC SIN X. Prepared by the Mathematical Tables Project conducted under the sponsorship of the National Bureau of Standards. 121 pages, 19 × 27 cms. New York, Columbia University Press, 1945. Price $3.50.

This is a 12 place table of values in radian measure which may also be used for obtaining the inverse cosine. The function is tabulated at intervals of 0.0001 in the range between 0 and 0.9890 at intervals of 0.00001 in the range between 0.98900 and unity. Means are provided for interpolation. It is stated that the entries are believed to be correct to within 0.51 units in the last place.

R. H. OPPERMANN.

THERMODYNAMIC PROPERTIES OF AIR, by Joseph H. Keenan and Joseph Kaye. 73 pages, tables, 19 × 26 cms. New York, John Wiley & Sons, Inc. London, Chapman & Hall, Limited, 1945. Price $2.25.

It is stated in the preface of this work that the need for a working table of the thermodynamic properties of air has been emphasized recently by the rapidly growing interest in the gas turbine. This book consists of a series of thirteen tables which should meet all but the most exacting requirements for calculations of gas turbine processes. Table I, on air at low pressures, gives for temperatures between 300 and 6,500° F. abs. the magnitudes of enthalpy, relative pressure, internal energy, relative volume and ϕ for dry air. Succeeding tables cover R Log e N; thermodynamic properties, values for viscosity, thermal conductivity, Prandth number; polytropic functions of pressure ratio, $n = 1.4$, etc.

In the back, the sources of information and methods used are given with a set of examples in the use of the tables, and a bibliography.

R. H. OPPERMANN.

ATOMIC ENERGY FOR MILITARY PURPOSES. The Official Report on the Development of the Atomic Bomb under the Auspices of the United States Government, 1940–1945, by Henry DeWolf Smyth. IX + 264 pages, plates, Princeton, Princeton University Press, 1945. Price, paper $1.25, cloth $2.50.

When one glances through Professor Smyth's book, "Atomic Energy for Military Purposes," one is surprised by several considerations—the amount of detailed information given in it, the clarity with which it has been possible to present the information, and the possibility of producing the book such a short time subsequent to the period when the intensive activities on the atomic bomb itself must have left very little leisure for writings of this kind.

For a book of this nature it is necessary to give the more or less historical details of the development of the project in its administrative and political aspects. This might easily have resulted in something which was tiresome to read. However, Professor Smyth has moulded these matters so cleverly with the general discussion of the problem that they have assumed a real and vital interest and, indeed, by presenting the developmental picture at various stages, they have really served to clarify the scientific understanding of the project to the reader. The summaries at the end of the various chapters also serve a very useful purpose in crystalizing the essentials in the mind of the reader and removing certain cobwebs with which his thinking may have been encumbered in the general reading.

The book starts out with an exceptionally clear presentation of such fundamentals of nuclear physics as are necessary to understand what is written subsequently. Then comes a discussion of the need of a chain reaction and a statement of the meaning of fission. In the second chapter we encounter an explanation of the various essential elements which enter into

an attempt to realize the chain reaction. The author keeps our minds and hopes in the stage of development which probably characterized the research group itself during that period. Throughout the book the thoughts of the reader are allowed to develop in the sequence that thoughts probably developed in the project itself, and pedagogically this principle has played no small part in maintaining the clarity of the book throughout. The reader is always in a state of expectation as to what is going to happen next. His mind is a little ahead of his reading, and he is delighted to see what he has thought of discussed as the book proceeds. In the third chapter, we have the story of some of the initial actions which were instrumental in launching the problem. The fourth chapter leads us further into the problems whose solution was necessary to decide whether a chain reaction could be possible. In the fifth chapter, we read of the administrative history at a time when the Government had come to take a real interest in the project as something likely to attain practical success. In the sixth chapter we attain the primary goal of realizing the first self-sustaining pile. The seventh and eighth chapters deal largely with the development of the project in the direction of the production of plutonium. We see the project grow in size, we see the development of its administrative responsibilities, and we learn of the magnitude of the undertaking to secure proper health protection in the light of the radioactive products involved. Chapters 9, 10 and 11 deal with various methods of isotope separation which were tried with varied degrees of success as corrolaries to the main project. Chapter 12 is concerned with the work on the atomic bomb itself and with the essentials necessary to secure the right kind of explosion. Chapter 13 is a general summary. Five appendices dealing with technical matters follow. All that has been written up to this point is written in the spirit of expectation and before the actual realization of the test performed in New Mexico on July 16, 1945. This test is described in the last appendix No. 6, and the book ends before the use of the atomic bomb in Japan itself.

In a sense, one cannot but be surprised and gratified at the amount of real information which has been released in this work and, considering the various discussions which take place regarding the supposed secrets involved, it is difficult to see how, in the light of what is already known, any responsible group of physicists could be delayed for any appreciable time in realizing the essentials of pile operation and even bomb construction if they set their minds to do it.

W. F. G. SWANN.

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THE AMERICAN ANNUAL OF PHOTOGRAPHY, 1946, edited by Frank R. Fraprie and Franklin I. Jordan, 200 pages, illustrations, 18 × 25 cms. Boston, American Photographic Publishing Company, 1946. Price $2.00 (paper); $3.00 (cloth).

This long-lived annual appears in its sixtieth edition and proves that time has not dulled its capacity to interest and inform. The thirteen articles cover a variety of topics, practical technical and historical in approach. Yousuf Karsh, in the lead article "Superlative Personalities" discusses with succinct comments some of the notable personages he has photographed. The accompanying illustrations pay eloquent tribute to Mr. Karsh's ability.

Three authors have written of their experiences in nature photography, with birds, snow and trees. The silk screen process, flash lighting, and the stereoscope are the subjects of other papers. The section of pictorial illustrations continue to offer a selection of well chosen prints. Mr. Fraperie has added informative notes on the circumstances under which each picture was taken, as well as including technical data. "Who's Who in Pictorial Photography" is a continuation of a feature in previous volumes.

These various features unite to form a volume of interest to any camera fan.

GEORGE E. PETTENGILL.

PUBLICATIONS RECEIVED.

Waveform Analysis, A Guide to the Interpretation of Periodic Waves, Including Vibration Records, by R. G. Manley. 275 pages, tables and illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd. 1945. Price $4.00.

Weiss Magnetons as Components of Nuclear and Subnuclear Structures, by Theodore van Schelven. 32 pages, drawings, 16 × 24 cms. Amsterdam (Holland) Kosmos Publishing Company, 1945. Price $3.00 (paper).

The American Annual of Photography, Volume 60, 1946, edited by Frank R. Fraprie and Franklin I. Jordan. 200 pages illustrations, 18 × 25 cms. Boston, American Photographic Publishing Company, 1946. Price $2.00 (paper), $3.00 (cloth).

Mechanics for Engineers, by Edward R. Maurer, Raymond J. Roark, George W. Wsha. 425 pages, illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd., 1945. Price $4.00.

A Benjamin Franklin Reader, edited by Nathan G. Goodman. 818 pages, 15 × 22 cms. New York, Thomas Y. Crowell Company, 1945. Price $3.50.

The Milky Way, by Bart J. Bok and Priscilla F. Bok. 2nd Edition. 224 pages, illustrations, 15 × 22 cms. Philadelphia, The Blakiston Company, 1945. Price $2.50.

Atomic Energy in War and Peace, by Gessner G. Hawley and Sigmund W. Leifson. 211 pages, illustrations 13 × 20 cms. New York, Reinhold Publishing Corporation, 1945. Price $2.50.

CURRENT TOPICS.

ARMY AND NAVY NOTES.

Shortwave Radio is Key to Post War Progress.—Optimistic forecasts of expanded post war shortwave radio activities—in frequency modulation, television, international communications and high frequency heating for industrial processing and manufacture—are made by Walter Evans, Vice President in charge of radio, radar and electronics activities of the Westinghouse Electric Corporation, Baltimore, Maryland.

Greatest single factor contributing to this advance, he said, will be the vastly improved "know how" acquired in this promising field during the industry's record war production.

This forecast came on the twenty-first anniversary of shortwave's "coming of age," as Mr. Evans put it, pointing out that although the science had been known many years earlier, it was not until June 1924 that it attained general acceptance among world radio authorities.

That recognition was won, he recalled, in a dramatic demonstration by the late Dr. Frank Conrad, Assistant Chief Engineer of Westinghouse and one of a group of Americans attending a conference of international communications magnates meeting in London to consider a radio link between Europe and South America.

After lengthy discussions of an ultra-long wave link, Dr. Conrad invited several delegates—among them a former ship's wireless operator—to his hotel room where, using the curtain rod as an antenna, he had the operator copy telegraph news sent by shortwave from Pittsburgh. Informed next day by the still-amazed operator-delegate of the sensational test, the conference shortly thereafter decided to build a shortwave link and out of this recognition came the general acceptance responsible for all modern shortwave radio.

DEVELOPMENT HASTENED BY WAR.

Turning to war and post war uses, Mr. Evans continued: "Without shortwave we would have no radar, that near-magic development of the war which safeguards ships and planes from surprise attack and enables them to track down enemy craft; no static-free FM radio; no television; no low-power long distance communications; and no dielectric heating which today bonds plywood for PT boat hulls and serves a hundred other military and civilian uses.

"Each of these arts was known before the war and limited development was under way but it remained for the war's urgency to hasten their refinement and broaden their applications. Advances of two normal decades have been packed into a half dozen years of war and preparations for war and as a result V-J Day will find us with almost inexhaustable electronics 'know how' waiting to be harnessed to peacetime tasks."

Pointing out that electronics advances will bring not only the devices for better living, but by its widespread employment, purchasing power to afford and enjoy them, Mr. Evans declared:

"Frequency modulation radio service, an accomplished fact before the war, will be expanded to bring this noiseless reception to listeners in every metropolitan center across the land.

Radar Work Aids Television

"Our war-paced engineering and production of radar will yield proportionate advantage for television, its scientific first cousin. All answers for a completely satisfactory black-and-white television service already are at hand and war-learned lessons will speed development of improved color television."

Mr. Evans sees shortwave playing an increasingly important part in world affairs after victory with international shortwave stations fostering mutual understanding and good will among nations.

"The war has taught us that these long-range stations, which know no barriers of geographical frontiers or racial prejudice, can become powerful adjuncts of every nation's State Department or Foreign Office. Hitler and Hirohito demonstrated their maximum abuse. It is up to post war planners to shape this force to maximum good among nations.

High frequency heating, the electronics science's newest contribution to better-quality-at-lower-cost manufacture, also will reflect war developments in its post war applications, Mr. Evans said.

"Aided most by shortwave development will be its dielectric applications which have to do with non-conductor materials," he pointed out. "Already used to bond plywood, cure plastics and dry nylon yarn, this newest tool of industry will find hundreds of new opportunities to improve production and reduce costs for post war manufacturers.

"Also benefiting from the lessons of war production will be induction heating which is not dependent upon ultra-high frequency operation. This wonder process which has been reflowing tin, at a saving of up to 65 per cent. of this war-scarce material, will provide new and dependable manufacturing shortcuts in heat treating metals, annealing electrical steels, brazing and welding and many other essential shop operations."

R. H. O.

X-Rays Alter Wave Length, Change Color of Gems. (*Electrical Engineering*, Vol. 64, No. 7.)—X-rays and other types of radiation, including cathode or electronic rays and deuteron beams from a cyclotron, when allowed to pass through plates cut from crystalline substances, alter the mechanical strength of the material. There are accompanying changes in the color of the crystals, and the chemical reactivity also may be altered.

These discoveries of Clifford Frondel, former senior physicist of the United States Signal Corps and research associate at Harvard University, Boston, Mass., were recently demonstrated at the Reeves-Ely Laboratories, New York, N. Y.

It was explained that the X-rays cause an interchange of electrons between the atoms composing the crystal and, by altering the interatomic bonding forces, change the elastic constants of the material. By this irradiation it is possible to regulate the frequency of the quartz crystals used in radio and radar.

Millions of tiny plates of crystalline quartz are used by the armed services as oscillator plates to control radio communications. The frequency at which

the radio will transmit or receive is controlled in common types of crystals by the thickness of the plate, brought to proper thickness by a delicate operation. By using the new X-ray irradiation technique, oscillator plates can now be adjusted rapidly and easily to a desired frequency with a precision hitherto impossible.

Another interesting application of X-rays is in the altering or the increasing in intensity of gem stones, and some colorless stones of little value may be given intense hues. Diamonds have been colored green and golden brown by irradiation with deuterons.

Irradiation with X-rays also has been found to modify greatly the rate of the solution and the chemical reactivity of crystals, a fact which may open up a new field of X-ray photochemistry.

R. H. O.

Wartime Cotton Developments to Find Peacetime Uses.—Wartime research projects on cotton at the Southern Regional Research Laboratory have resulted in developments that will find important peacetime applications according to officials of the U. S. Department of Agriculture's research administration.

Among the results of research to develop an improved cotton tire cord are a cord with improved heat resistance, and another with greater tensile strength than is obtained with present commercial cotton tire cord processes. The investigators declare without qualification that cotton cords can be manufactured which are definitely superior to the types of commercial cotton cords now being produced, and further improvements can be expected as more information is obtained and applied.

Another wartime development at the laboratory that will find important peacetime uses is a process for rotproofing cotton goods that is more effective than any other known treatment. Cloth samples treated by the new process have retained more than 80 percent of their strength after burial for an entire year in a damp, warm soil known to be rich in the organisms that cause rotting. In contrast, untreated goods in the same soil rotted to pieces in about a week.

The investigators at the southern laboratory have also developed a flameproofing treatment for cotton fabric that has definite advantages. Although effective treatments have been available for a number of years, usually they have made the fabric too stiff and heavy or washed out too readily. Fabrics treated by the new process retain their flame resistance after many launderings, yet their physical characteristics are changed but little. The process makes flameproofing of outer garments more practicable.

Still other recent discoveries of the New Orleans laboratory are a resinous treatment that increases by many times the life of cotton fishnets, a specially treated cotton thread that will resist deterioration resulting from presence of acid in fertilizer bags, and a new type of semi-elastic all-cotton bandage. The latter, announced earlier in the war, has undergone extensive hospital trials and has been declared by medical men to be superior to ordinary gauze, particularly for elbow, knee, and head dressings.

R. H. O.